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PART I

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## UNSTEADY AERODYNAMICS FOR ADVANCED CONFIGURATIONS

### PART I—APPLICATION OF THE SUBSONIC KERNEL FUNCTION TO NONPLANAR LIFTING SURFACES

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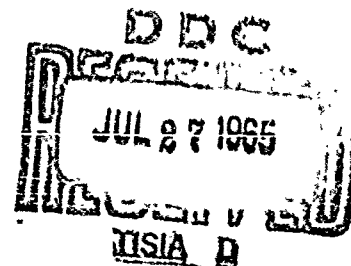
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RESEARCH AND TECHNOLOGY DIVISION  
AIR FORCE SYSTEMS COMMAND  
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

Project No. 1370, Task No. 137003



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The Space and Information Systems Division  
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## FOREWORD

This report covers the research conducted by the Space and Information Systems Division of North American Aviation, Inc., Downey, California, for the Aerospace Dynamics Branch, Vehicle Dynamics Division, Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio, under Contract No. AF 33(657)-10399.

The work was performed to advance the state of the art of flutter prevention for flight vehicles as part of the Research and Technology Division, Air Force Systems Command's exploratory development program. This research was conducted under Project No. 1370 "Dynamic Problems in Flight Vehicles," and Task No. 137003, "Prediction and Prevention of Dynamic Aerothermoelastic Instabilities." Mr. James Olsen of the Vehicle Dynamics Division, Air Force Flight Dynamics Laboratory was the Project Engineer.

Mr. L. V. Andrew was the Program Manager for North American Aviation. Mr. H. T. Vivian, under the guidance of Dr. H. Ashley, laid out the form of the solution and wrote the computer program. Dr. E. R. Rodemich made many significant contributions to the numerical analysis.

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This technical documentary report has been reviewed and is approved.

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## ABSTRACT

In this part, equations for pressure distributions and generalized aerodynamic forces are derived for a thin nonplanar lifting surface in simple harmonic motion at subsonic speeds. A digital computer program, written in Fortran IV, is also presented herein. The computer program will generate up to a ten by ten matrix of generalized aerodynamic forces when given data for the geometry of a planar lifting surface with a folded planar tip, the flight Mach number, the reduced frequency of motion, and some control constants. Control surface deflections are not accounted for in this study.

The kernel function method given by Watkins, Runyan, and Woolston (Reference 1), which relates the pressure distribution to the downwash on a planar lifting surface, has been extended and applied to a nonplanar lifting surface. Hsu's technique (Reference 4) of employing Gaussian quadrature formulas is used when integrating the product of the kernel function and the lift function over the planform area.

Recommendations are made to extend the method to account for blunted leading edges and the accompanying airfoil thickness and to account for control surface deflections.

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# SYMBOLS

|                 |  |
|-----------------|--|
| $a$             | Local speed of sound (constant in linear theory)                                   |
| $b_o$           | Root semichord   |
| $b(s)$          | Local semichord  |
| $C_p$           | Pressure coefficient   |
| $\bar{C}_p$     | Time independent factor of oscillatory part of $C_p$                               |
| $i, j, k$       | Unit vectors parallel to the coordinate axes                                       |
| $i$             | $\sqrt{-1}$  |
| $k$             | Reduced frequency, $\omega b_o / U_\infty$   |
| $k_1$           | $kr_1$   |
| $K$             | Kernel function  |
| $K_0, K_1, K_2$ | Modified Bessel functions of the second kind                                       |
| $M$             | Local Mach number (same as free stream Mach number, $M_\infty$ , in linear theory) |
| $n$             | Coordinate measured normal to wing surface   |
| $n(x, s, t)$    | Contribution to $n$ of elastic deformation of the wing                             |
| $n_t(x, s)$     | Contribution to $n$ of wing thickness  |
| $\bar{n}(x, s)$ | Time independent factor of $n(x, s, t)$  |
| $p$             | Pressure   |
| $\vec{q}$       | Fluid velocity   |
| $R$             | Gas constant   |
| $R$             | $\sqrt{(x-\xi)^2 + \beta^2 r_1^2}$   |

List of Symbols continued on next page.

|                    |   |
|--------------------|---|
| $r_1$              | $\sqrt{(y-\eta)^2 + (z-\zeta)^2}$   |
| $s$                | Curvilinear coordinate on the wing (page 10)  |
| $T$                | Absolute temperature  |
| $t$                | Time  |
| $u, l$             | Subscripts indicating upper and lower wing surfaces   |
| $u, v, w$          | Components of perturbation velocity   |
| $U, W$             | x- and z-components of $\vec{V}$  |
| $U_\infty$         | $ \vec{V} $ (speed at infinity)   |
| $\vec{V}$          | Uniform fluid velocity at infinity  |
| $\bar{W}$          | Time independent factor of normal velocity  |
| $x, y, z$          | Cartesian coordinates   |
| $x_0$              | $x - \xi$   |
| $y_0$              | $y - \eta$  |
| $\beta$            | $\sqrt{1-M^2}$  |
| $\gamma$           | Ratio of specific heats   |
| $\gamma(s)$        | Local angle between wing surface and xy-plane   |
| $\Delta \bar{p}$   | Time independent factor of pressure difference between wing surfaces, $\bar{p}_l - \bar{p}_u$ |
| $\xi, \eta, \zeta$ | Cartesian coordinates   |
| $\bar{\xi}$        | Coordinate on the wing (page 22)  |
| $\phi$             | Velocity potential  |
| $\varphi$          | Perturbation velocity potential   |
| $\bar{\varphi}$    | Time independent factor of $\varphi$  |
| $\bar{\psi}$       | Acceleration potential  |
| $\rho$             | Fluid density   |
| $\omega$           | Angular frequency (radians per unit time)   |

## I. INTRODUCTION

The first published numerical method for solving the subsonic pressure distribution problem for planar lifting surfaces undergoing simple harmonic motion was developed at NASA's Langley Research Center by Watkins, Runyan, and Woolston (Reference 1). Watkins, et al., presented two methods of handling the numerical integration of the kernel function in the region where high-order singularities exist. Both methods involved a dense concentration of integration points in the neighborhood of the singularity. Using these methods, it is possible to obtain downwash integrals, in terms of the pressure-loading coefficients, at any arbitrary set of points on the surface (e. g., at all the kinematic downwash points known from previously determined vibration mode data). However, in order to reduce the running time on the computer, the downwash integrals were obtained at a selected set of collocation points, such as those at intersections of quarter, half, and three-quarter chord stations and like half-span stations. When downwashes were matched exactly (and thus, boundary conditions) at these collocation points, responsibility was placed upon the user to evaluate the kinematic downwashes there. A least-square error surface fitted to the mode data was commonly used to evaluate them. Furthermore, if the user desired that the boundary conditions be satisfied at a greater number of points, it was necessary that he use a correspondingly greater number of loading functions.

Procedures were then described by Rodden and Revell (Reference 2) and the correct form of the equations were presented by Fromme (Reference 3) for calculating pressure-loading coefficients which match a greater number of kinematic downwashes than coefficients, in the sense that the sum of squares of amplitudes of differences of complex numbers are minimized. Since it was still the responsibility of the user to evaluate the kinematic downwashes at the collocation point, least-square error procedures were used twice: once implicitly and once explicitly.

Hsu (Reference 4) significantly advanced the logical development of the kernel function approach when he established an optimum set of collocation and integration points. He started with the previously established chordwise pressure functions based on steady-state, two-dimensional, incompressible aerodynamics, and with spanwise loading functions, based on steady-state lifting-line theory. He concluded that there is sufficient reason to believe that these functions display the proper characteristics near the edges of lifting surfaces oscillating in a compressible fluid.

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Returning to the two-dimensional case, Hsu established that if the chordwise distribution of modal deflections (and thus downwashes) is accurately represented by a polynomial of degree  $2N-1$  and is approximated by a polynomial of degree  $N-1$ , then the integral for the sectional load is evaluated with zero error by a  $N$ -point Gaussian quadrature if the difference between the accurate and the approximate representation of the downwashes is made equal to zero at each of the  $N$  points (i. e., the chordwise collocation stations). Conversely, still for the two-dimensional case, Hsu established that if the product of the pressure function and the kernel, divided by the Jacobi-Gauss weight factor (which produces the square root singularity at the leading edge), is accurately represented by a polynomial of degree  $2N-1$ , then the integral for the downwash at any one of the collocation stations is evaluated with zero error by a  $N$ -point Gaussian quadrature. These  $N$  points are then made the chordwise integration stations.

For the spanwise direction, using lifting-line theory, Hsu similarly established  $M$ -spanwise collocation stations and  $M + 1$  interdigitated spanwise integration stations plus the conditions under which the Gaussian quadrature can be used with zero error.

It is important to note that the kernel of the integral equation for the downwashes in unsteady, three-dimensional, compressible flow cannot be accurately represented by a polynomial of finite degree. It is equally important to note, however, that, because of the edge characteristics of the pressure and loading functions, the Gaussian quadratures employed at Hsu's optimum point set evaluate the integrals with the least squared error for a given number of integration points. We have yet to match the boundary conditions using Hsu's method.

The downwash matching problem in Hsu's approach is basically the same as in Watkin's approach; we merely have a more logical choice of points at which to match them. In the examples Hsu used to demonstrate his approach, he chose to use the same number of pressure-loading functions as collocation points. However, the approach is not dependent upon that choice. If a smaller number of pressure-loading functions are used, then the procedures described by Rodden, Revell, and Fromme may be used to compute pressure-loading coefficients which yield a minimum sum of squares of amplitudes of differences in downwashes.

A need has arisen for application of the kernel function method to non-planar lifting surfaces on future aerospace vehicles. Application is also required to more conventional non-planar surfaces such as T-tail, V-tail, and wing-vertical tail combinations.

Professor H. Ashley outlined the application to the folded tip configuration. A computer program based on Ashley's work was developed for steady-state flow by L. Johnson, et al., of the Los Angeles Division of North American Aviation, Inc.

The work reported herein is based on Professor Ashley's outline. However, the expression for the kernel has been greatly simplified by Dr. E. R. Rodemich of North American Aviation, Inc., Space and Information Systems Division.

## II. FUNDAMENTAL EQUATIONS OF FLUID MOTION

Consider a body immersed in a compressible, nonviscous, perfect fluid and assume the fluid flow to be isentropic and irrotational. Under these conditions, a velocity potential  $\phi$ , exists:

$$\vec{q} = \nabla \phi \quad (1)$$

where  $\vec{q}$  is the velocity vector of a fluid element and  $\nabla$  is the gradient operator (See Reference 4.) Also under these conditions, the isentropic (constant entropy) pressure-density relationship is valid. Thus,

$$a^2 = \left( \frac{\partial p}{\partial \rho} \right)_s = \gamma R T \quad (2)$$

Other equations which govern the flow are the continuity equation for conservation of mass

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{q}) = 0 \quad (3)$$

and Euler's equations for conservation of momentum

$$\frac{D\vec{q}}{Dt} = -\frac{1}{\rho} \nabla p \quad (4)$$

where,  $\frac{D}{Dt}$ , the substantial derivative, is

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{q} \cdot \nabla \quad (5)$$

These equations may be combined, as described in Reference 5, to yield the nonlinear, unsteady flow equation

$$\nabla^2 \phi - \frac{1}{a^2} \left[ \frac{\partial^2 \phi}{\partial t^2} + \frac{\partial q^2}{\partial t} + (\vec{q} \cdot \nabla) \frac{q^2}{2} \right] = 0 \quad (6)$$

Consider, then, that the fluid motion consists of a perturbation superimposed on a uniform stream velocity  $\vec{V} = U\vec{i} + W\vec{k}$  parallel to the  $xz$ -plane of a rectangular Cartesian coordinate system. Then the velocity potential may be expressed as the sum of a uniform part and a perturbation part

$$\phi = Ux + Wz + \varphi \quad (7)$$

and, similarly, the velocity vector becomes

$$\vec{q} = \vec{V} + \nabla\varphi = \nabla\phi \quad (8)$$

The pressure coefficient at any point in an isentropic flow field is

$$C_p = \frac{p - p_\infty}{\frac{1}{2} \rho_\infty U_\infty^2} \quad (9)$$

where  $p - p_\infty$  is the difference between local pressure and free-stream pressure,  $U_\infty = |\vec{V}|$ , and  $1/2 \rho_\infty U_\infty^2$  is the free-stream dynamic pressure. From Kelvin's equation (Reference 5) for isentropic flow

$$C_p = \frac{2}{\gamma M_\infty^2} \left\{ \left[ 1 + \frac{\gamma - 1}{2} M_\infty^2 \left( 1 - \frac{\vec{q} \cdot \vec{q} + 2 \frac{\partial \varphi}{\partial t}}{U_\infty^2} \right) \right]^{\frac{\gamma}{\gamma - 1}} - 1 \right\} \quad (10)$$

A complete statement of the fundamental problem requires specification of the boundary conditions. The boundary conditions at infinity depend upon the free-stream velocity. When it is less than the speed of sound in the fluid, the disturbances to the flow die out and are not felt at infinity. When it is greater than the sonic speed, then in the region where disturbances are felt, even at infinity, the component of flow due to the disturbance is directed away from the source of disturbance and otherwise the free-stream flow is undisturbed. The boundary conditions at the surface of the body require that the flow be tangent to the surface everywhere on the body. This condition is satisfied by the equation

$$\frac{D}{Dt} \psi(x, y, z, t) = 0 \quad (11)$$

where

$$B(x, y, z, t) = 0 \quad (12)$$

is the equation for the position of the surface at any time  $t$ , and the substantial derivative  $D/Dt$  is defined by Equation 5.

### III. LINEARIZED EQUATIONS OF MOTION

Linearization of the equations of motion is not dependent upon an explicit form of the body equation, Equation 12, so long as the normal derivatives of the equation are everywhere nearly perpendicular to the free-stream direction. Thin lifting surfaces at small angle of attack satisfy this condition and are treated herein and in Parts 2 and 4 of this report. The special considerations required for thick bodies and high angles of attack are treated in Parts 3 and 5. The following development is, therefore, restricted to thin airfoils.

We first obtain the specialized form of Equation 7 when the uniform stream velocity lies along the x-axis; i. e.,  $W = 0$  and, therefore,  $\vec{V} = U\vec{i}$ ,  $U_\infty = U$ . The velocity potential is

$$\phi = Ux + \varphi \quad (13)$$

and the velocity vector of a fluid element becomes

$$\vec{q} = (U + u)\vec{i} + v\vec{j} + w\vec{k} \quad (14)$$

where

$$u = \frac{\partial \varphi}{\partial x}, \quad v = \frac{\partial \varphi}{\partial y}, \quad \text{and} \quad w = \frac{\partial \varphi}{\partial z}$$

The perturbation velocities  $u$ ,  $v$ , and  $w$  are assumed to be much smaller than the free-stream velocity; i. e.,  $u, v, w \ll U$ .

The linearization procedure when applied to Equation 10 yields the fully linearized pressure coefficient

$$C_p = -\frac{2}{U_\infty^2} \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) \varphi \quad (15)$$

and when applied to Equation 6 yields the fully linearized unsteady flow equation

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} - \frac{1}{a_\infty^2} \left[ U^2 \frac{\partial^2 \varphi}{\partial x^2} + 2U \frac{\partial^2 \varphi}{\partial x \partial t} + \frac{\partial^2 \varphi}{\partial t^2} \right] = 0 \quad (16)$$

Next, we write the body equation for a thin, nonplanar lifting surface (Figure 1), in terms of a curvilinear coordinate

$$s = s(y)$$

$s$  represents the integral of distance along the line of the mean position of the airfoil from the centerline to  $y$ ,

$$s(y) = \int_0^y ds \quad (17)$$

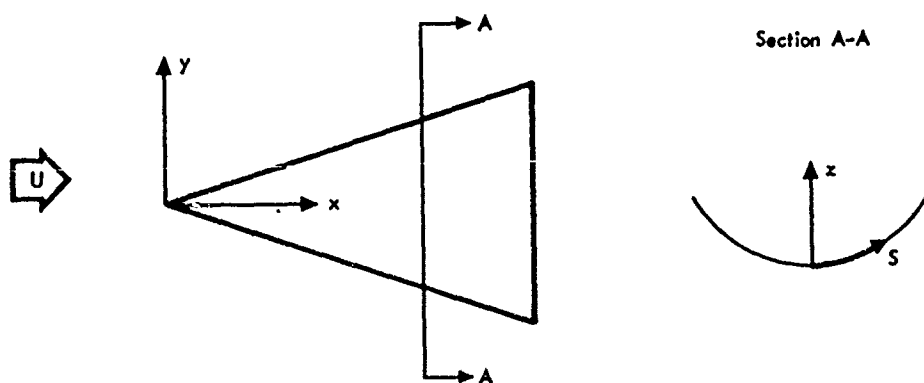


Figure 1. Generalized Curvilinear Planform

The position of the surface in terms of  $s$  and  $n$  (the normal to  $S$ ), is separated into two parts; one part for the upper (or inner) surface, and the other for the lower (or outer) surface

$$B_u(x, n, s, t) = n - n_\tau(x, s) - n(x, s, t) \quad (18a)$$

$$B_l(x, n, s, t) = n + n_\tau(x, s) - n(x, s, t) \quad (18b)$$

where  $n_\tau(x, s)$  represents the thickness of the airfoil and  $n(x, s, t)$  represents the elastic deflection of the airfoil. In accordance with Equation 11, we use the operator

$$\nabla = i \frac{\partial}{\partial x} + j' \frac{\partial}{\partial s} + k' \frac{\partial}{\partial n} \quad (19)$$

on Equation 18a to get

$$\begin{aligned} \nabla B_u(x, n, s, t) = & -i \left[ \frac{\partial}{\partial x} n_\tau(x, s) + \frac{\partial}{\partial x} n(x, s, t) \right] \\ & - j' \left[ \frac{\partial}{\partial s} n_\tau(x, s) + \frac{\partial}{\partial s} n(x, s, t) \right] + k' \end{aligned}$$

Substitution into

$$\frac{D}{Dt} B_u(x, s, n, t) = \frac{\partial B_u}{\partial t} + (U i + \nabla \varphi) \cdot \nabla B_u = 0 \quad (20)$$

of the equation for the upper surface, after higher order terms have been discarded, gives

$$\frac{\partial \varphi}{\partial n} = \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) n(x, s, t) + U \frac{\partial}{\partial x} n_\tau(x, s) \quad (21)$$

The same procedure, for the lower surface, gives

$$\frac{\partial \varphi}{\partial n} = \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) n(x, s, t) - U \frac{\partial}{\partial x} n_T(x, s) \quad (22)$$

Finally, we restrict the analysis to that class of problems in which the effects of thickness on the time dependent forces can be neglected. By letting

$$n_T(x, s) = 0 \quad (23)$$

we get a single expression for the boundary condition

$$\frac{\partial \varphi}{\partial n} = \left( \frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right) n(x, s, t) \quad (24)$$

It is evident from Equation 24 that, when the motion of the surface is simple harmonic motion,

$$n(x, s, t) = \bar{n}(x, s) e^{i\omega t} \quad (25)$$

then,

$$\varphi(x, s, n, t) = \bar{\varphi}(x, s, n) e^{i\omega t} \quad (26)$$

Substitution of Equations 25 and 26 into Equations 15, 16, and 24 gives

$$\bar{C}_p = - \frac{2}{U_\infty^2} \frac{D\bar{\varphi}}{Dt} \quad (27)$$

$$\nabla^2 \bar{\varphi} = \frac{1}{a_\infty^2} \frac{D^2 \bar{\varphi}}{Dt^2} \quad (28)$$

$$\frac{\partial \bar{\varphi}}{\partial n} = \frac{D\bar{n}}{Dt} \quad (29)$$

where

$$\frac{D}{Dt} \equiv U \frac{\partial}{\partial x} + i\omega \quad (30)$$

and

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial s^2} + \frac{\partial^2}{\partial n^2} \quad (31)$$

To this degree of approximation,  $M = M_\infty$  and  $a = a_\infty$ .

#### IV. THE ACCELERATION POTENTIAL

##### PLANAR WINGS

Equation 27 shows that the calculation of pressure requires taking derivatives of the velocity potential, and Equation 29 states the boundary condition which must be satisfied. Watkins, Runyan and Woolston (Reference 1) solved this problem for planar surfaces in terms of a series of pressure functions, or acceleration potential functions ( $\bar{\psi}$ ), where,

$$\bar{\psi}(x, y, z) = \frac{D}{Dt} \bar{\phi}(x, y, z) \quad (32)$$

Integration of Equation 32 gives the general expression for the velocity potential in terms of the acceleration potential:

$$\bar{\phi}(x, y, z) = \frac{1}{U} e^{-i\omega \frac{x}{U}} \int_{-\infty}^x e^{i\omega \frac{\lambda}{U}} \bar{\psi}(\lambda, y, z) d\lambda \quad (33)$$

if the velocity at infinity is  $\bar{V} = U\mathbf{i}$ . The velocity potential  $\phi$  due to a pulsating doublet satisfies Equation 28 (in the planar case  $s = y$  and  $n = z$ ); and, because the order of operators is interchangeable, the acceleration potential  $\psi$  also satisfies Equation 28.

A complete discussion of the application of boundary conditions in the planar case is given in Section 6-4 of Reference 5. The application in the nonplanar case is discussed in less detail in the following text.

##### NONPLANAR WINGS

In the nonplanar case, the acceleration potential at a point  $(x, s, N)$  due to a pulsating doublet located at the point  $(\xi, \sigma, n)$ , or  $(\xi, \eta, \zeta)$  in the direction of  $n$  is

$$\bar{\psi}(x, s, N) = -A \frac{\partial}{\partial n} \left[ \frac{e^{i\omega \left[ \frac{M}{a_{\infty}^2} (x-\xi) - \frac{R}{a_{\infty}^2} \right]}}{R} \right] \quad (34)$$

where

$$\frac{\partial}{\partial n} = \cos \gamma (\eta) \frac{\partial}{\partial \zeta} - \sin \gamma (\eta) \frac{\partial}{\partial \eta} \quad (35)$$

$$R = \sqrt{(x - \xi)^2 + \beta^2 [(y - \eta)^2 + (z - \zeta)^2]} \quad (36)$$

and  $\gamma(\eta)$  is angle between the wing and the xy-plane at the point  $(\xi, \eta, \zeta)$ . The perturbation velocity potential may be built up from a distribution of doublets of acceleration potential over the wing. If  $A$  is the infinitesimal doublet strength at  $(\xi, \sigma, \eta)$ , the contribution to  $\bar{\phi}$  from the doublet at this point, from Equations 33 and 34, is

$$\Delta \bar{\phi}(x, s, N) = \frac{-A}{U} \frac{\partial}{\partial n} e^{-i\omega \frac{x - \xi}{U}} \int_{-\infty}^{x - \xi} e^{\frac{i\omega \left[ \frac{\lambda}{U} + \frac{M\lambda}{a_\infty^2 \beta^2} - \frac{R'}{a_\infty^2 \beta^2} \right]} } \frac{d\lambda}{R'} \quad (37)$$

where

$$R' = \sqrt{\lambda^2 + \beta^2 [(y - \eta)^2 + (z - \zeta)^2]}$$

and the velocity component normal to the surface at  $(x, s, N)$  is

$$\Delta \bar{w}(x, s, 0) = \lim_{N \rightarrow 0} \frac{\partial}{\partial N} \Delta \bar{\phi}(x, s, N) \quad (38)$$

where

$$\frac{\partial}{\partial N} = \cos \gamma (y) \frac{\partial}{\partial z} - \sin \gamma (y) \frac{\partial}{\partial y} \quad (39)$$

Note that when the operator  $\partial/\partial n$  is applied in Equation 37, the partials  $\partial/\partial \zeta$  and  $\partial/\partial \eta$  may be replaced by  $-\partial/\partial z$  and  $-\partial/\partial y$ , respectively. Substitution of Equation 37 into Equation 38 gives

$$\Delta \bar{w}(x, s, 0) = \frac{A}{U} e^{-i\omega \frac{x - \xi}{U}} \lim_{N \rightarrow 0} P \int_{-\infty}^{x - \xi} e^{\frac{i\omega(\lambda - MR')/U\beta^2}{R'}} d\lambda \quad (40)$$

where the operator  $P$  is

$$P = \left[ \cos \gamma (y) \frac{\partial}{\partial z} - \sin \gamma (y) \frac{\partial}{\partial y} \right] \left[ \cos \gamma (\eta) \frac{\partial}{\partial z} - \sin \gamma (\eta) \frac{\partial}{\partial y} \right] \quad (41)$$

and finally

$$A = \frac{\Delta \bar{p} d\xi d\sigma}{4\pi \rho} \quad (42)$$

and  $\Delta \bar{p}$  is the complex amplitude of the difference in pressure on the upper and lower sides of the surface at  $(\xi, \sigma)$ ,

$$\Delta \bar{p}(\xi, \sigma) = \bar{p}_u(\xi, \sigma) - \bar{p}_l(\xi, \sigma) \quad (43)$$

and  $d\xi d\sigma$  is the incremental area of the doublet sheet.

The normal wash at  $(x, s, 0)$  given by Equation 40 is that due to a point pressure doublet at  $(\xi, \sigma, n = 0)$ . The total normal wash is the integral over the surface of all the pressure doublets,

$$\frac{\bar{w}(x, s, 0)}{U} = \frac{-1}{4\pi \rho U^2} \iint \Delta \bar{p}(\xi, \sigma) K(x - \xi, s, \sigma, \omega, M) d\xi d\sigma \quad (44)$$

where  $\iint$  denotes the Mangler formula for evaluating infinite integrals (Reference 7), and the kernel of the integral equation is (omitting the arguments  $\omega, M$  for brevity)

$$K(x_o, s, \sigma) = \lim_{\substack{n \rightarrow 0 \\ N \rightarrow 0}} \left\{ e^{-i \frac{\omega x_o}{U}} P \int_{-\infty}^{x_o} \frac{e^{i \omega (\lambda - MR')/U\beta^2}}{R'} d\lambda \right\} \quad (45)$$

where

$$R' = \sqrt{\lambda^2 + \beta^2 r_1^2}$$

$$r_1 = \sqrt{y_o^2 + z_o^2}$$

and

$$x_o = x - \xi, y_o = y - \eta, \text{ and } z_o = z - \zeta$$

Now, by putting  $k_1 = \omega r_1 / U$  and  $v = \lambda / \beta r_1$ ; then, by putting  $u = -\frac{v - M \sqrt{1+v^2}}{\beta}$  the integral in Equation 45 may be written

$$I_o \equiv \int_{-\infty}^{x_o} \frac{e^{i\omega(\lambda - MR')/U\beta^2}}{R'} d\lambda = \int_{u_1}^{\infty} \frac{e^{-ik_1 u}}{\sqrt{1+u^2}} du \quad (46)$$

where

$$u_1 = -\frac{x_o - MR}{\beta^2 r_1}$$

By breaking up the interval of integration into three subintervals and in the first two integrals letting  $u = w/i$

$$I_o = -i \int_0^1 \frac{e^{-k_1 w}}{\sqrt{1-w^2}} dw + \int_1^{\infty} \frac{e^{-k_1 w}}{\sqrt{w^2-1}} dw - \int_0^{u_1} \frac{e^{-ik_1 u}}{\sqrt{1+u^2}} du$$

or,

$$I_o = K_o(k_1) - i \int_0^1 \frac{e^{-k_1 w}}{\sqrt{1-w^2}} dw - \int_0^{u_1} \frac{e^{-ik_1 u}}{\sqrt{1+u^2}} du \quad (47)$$

where  $K_o(k_1)$  is the modified Bessel function of the second kind and zeroth order of the argument  $k_1$ .

To obtain the analytic form of the kernel, we write the operator  $P$  (Equation 41) in a more convenient form, taking advantage of the fact that  $I_o$  is a function of only  $r_1$  when  $x_o$  is held constant

$$PI_o = \cos [\gamma(y) - \gamma(\eta)] \left( \frac{1}{r_1} \frac{\partial I_o}{\partial r_1} \right) + [z_o \cos \gamma(y) - y_o \sin \gamma(y)] [z_o \cos \gamma(\eta) - y_o \sin \gamma(\eta)] \left( \frac{1}{r_1} \frac{\partial}{\partial r_1} \right) \left( \frac{1}{r_1} \frac{\partial I_o}{\partial r_1} \right)$$

The resulting kernel is

$$\begin{aligned}\bar{K}(x_o, s, \sigma) &= s_{TIP}^2 K(x_o, s, \sigma) \\ &= e^{-i \frac{\omega x_o}{U}} \left\{ \frac{T_1 K_1(x_o, s, \sigma) + T_2 K_2(x_o, s, \sigma)}{r_1^2} \right\} \quad (48)\end{aligned}$$

where

$$\begin{aligned}T_1 &= \cos [v(\underline{s}) - v(\underline{\sigma})] \\ T_2 &= \left[ \frac{z_o}{r_1} \cos v(s) - \frac{y_o}{r_1} \sin v(\underline{s}) \right] \left[ \frac{z_o}{r_1} \cos v(\underline{\sigma}) - \frac{y_o}{r_1} \sin v(\underline{\sigma}) \right] \quad (48a)\end{aligned}$$

$$\begin{aligned}K_1(x_o, s, \sigma) &= -k_1 K_1(k_1) - \frac{x_o}{R} e^{-k_1 u_1} + ik_1 \int_0^1 \frac{w e^{-k_1 w}}{\sqrt{1-w^2}} dw \\ &\quad + ik_1 \int_0^{u_1} \frac{u e^{-ik_1 u}}{\sqrt{1+u^2}} du \\ K_2(x_o, s, \sigma) &= k_1^2 K_2(k_1) + \left( \frac{2x_o}{R} + \frac{\beta^2 r_1^2 x_o}{R^3} + i \frac{k_1 x_o (Mr_1 + Ru_1)}{R^2} \right) e^{-ik_1 u_1} \\ &\quad - ik_1 \int_0^1 \frac{w e^{-k_1 w}}{\sqrt{1-w^2}} dw - ik_1^2 \int_0^1 \frac{w^2 e^{-k_1 w}}{\sqrt{1-w^2}} dw \\ &\quad - ik_1 \int_0^{u_1} \frac{u e^{-ik_1 u}}{\sqrt{1+u^2}} du + k_1^2 \int_0^{u_1} \frac{u^2 e^{-ik_1 u}}{\sqrt{1+u^2}} du \quad (48b)\end{aligned}$$

$$r_1 = s_{TIP} \sqrt{y_o^2 + z_o^2}$$

$$R = \sqrt{x_o^2 + \beta^2 r_1^2}$$

$$u_1 = -\frac{x_o - MR}{\beta^2 r_1}$$

$$k_1 = \frac{\omega r_1}{U} \quad (48c)$$

and  $K_1(k_1)$  and  $K_2(k_1)$  are modified Bessel functions of the second kind and first and second orders. The sub-bar indicates division by  $s_{TIP}$ , e.g.,  $\bar{r}_1 = r_1/s_{TIP}$ .

## V. THE BOUNDARY CONDITIONS

The remainder of the problem is to match the boundary conditions; i. e., to find a pressure-loading function  $\Delta \bar{p}(\xi, \sigma)$  which, when inserted into Equation 45 and integrated over the surface, yields the kinematic down-washes at selected points on the surface  $w(x, s, 0)$ .

In subsonic flow, the behavior of the pressure distribution is known in the area of the wing edges from a few of the exact solutions in lifting surface theory. In the neighborhood of the leading edge, the pressure should behave as

$$\lim_{\delta \rightarrow 0} \sqrt{\frac{1}{\delta}}$$

In the neighborhood of the trailing edge and all edges parallel to the free-stream direction, the pressure should behave as

$$\lim_{\delta \rightarrow 0} \sqrt{\delta}$$

where  $\delta$  is the distance to the wing edge. Both Hsu and Watkins employ a linear superposition of functions that satisfy these conditions. Hsu's function differs from Watkins only in that for any given number of terms in the series, Hsu's terms are linear combinations of Watkins terms. We use a normalized form of the function given by Watkins:

$$\Delta \bar{p}(\xi, \sigma) \equiv \frac{\rho U^2/2}{b(\sigma)} \sqrt{1 - \sigma^2} \sum_{n=0}^N \sum_{m=0}^M a_{nm} \sigma^m f_n(\xi) \quad (49)$$

where

$$f_0(\xi) \equiv \sqrt{\frac{1 - \xi}{1 + \xi}}$$

$$f_n(\xi) \equiv \sqrt{1 - \xi^2} U_n(\xi); 1 \leq n$$

$$U_1(\tilde{\xi}) = 1.0$$

$$U_2(\tilde{\xi}) = -2\tilde{\xi}$$

$$U_n(\tilde{\xi}) = -(2\tilde{\xi} U_{n-1} + U_{n-2}); 3 \leq n$$

and

$$\tilde{\xi} \equiv \left( \xi - \frac{\xi_{LE} + \xi_{TE}}{2} \right) / b(\sigma)$$

$$b(\sigma) = \frac{\xi_{TE} - \xi_{LE}}{2}$$

The  $a_{nm}$ 's are unknown pressure coefficients to be determined by matching the kinematic downwashes at the selected points  $(x_j, s_r)$  on the surface. Substitution of Equation 49 into Equation 44 leads to the matrix equation given by Rodden and Revell (Reference 2) Equation 39, for the point set  $x_j, s_r$

$$\left[ \frac{\bar{w}_i}{U} \right] = [D_{nm}^i] [a_{nm}] \quad (50)$$

where, in this case,

$$D_{nm}^i = \frac{1}{8\pi} \int_{-1}^1 \sqrt{1 - \sigma^2} \sigma^m \int_{-1}^1 f_n(\tilde{\xi}) \bar{K}(x_j - \xi, s_r, \sigma) d\tilde{\xi} d\sigma \quad (51)$$

We now reexamine the fundamentals of the problem before proceeding to evaluate the integrals in Equation 51 and thence to solve Equation 50.

One of the basic reasons for development of the kernel function method is that pressure distributions over a continuous lifting surface are smooth continuous functions that can be represented with reasonable accuracy by a series of analytic functions. We point out that  $f_n(\tilde{\xi})$  can be written

$$f_0(\tilde{\xi}) = \frac{1 - \tilde{\xi}}{\sqrt{1 - \tilde{\xi}^2}}$$

$$f_n(\tilde{\xi}) = \frac{1 - \tilde{\xi}^2}{\sqrt{1 - \tilde{\xi}^2}} U_n(\tilde{\xi}); 1 \geq n$$

and, therefore, the inner integral in Equation 51 may be written

$$I_1 = \int_{-1}^1 \frac{P_n(\tilde{\xi})}{\sqrt{1 - \tilde{\xi}^2}} d\tilde{\xi} \quad (52)$$

where

$$P_0(\tilde{\xi}) = (1 - \tilde{\xi}) \bar{K}(x_j - \tilde{\xi}, \underline{s}_r, \underline{\sigma})$$

$$P_n(\tilde{\xi}) = (1 - \tilde{\xi}^2) U_n(\tilde{\xi}) \bar{K}(x_j - \tilde{\xi}, \underline{s}_r, \underline{\sigma}); 1 \leq n$$

Now, we assume for the moment that the kernel  $\bar{K}(X_j - \xi, s_r, \sigma)$  can be represented with reasonable accuracy by a polynomial in  $\xi$ . Then,  $P_n(\tilde{\xi})$  is also a polynomial in  $\tilde{\xi}$ , and the Chebyshev-Gauss quadrature formula may be used to obtain the exact value of the integral expression (52); i. e.,

$$\int_{-1}^1 \frac{P_n(\tilde{\xi})}{\sqrt{1 - \tilde{\xi}^2}} d\tilde{\xi} = \sum_{k=1}^K \frac{\pi}{K} P_n(\tilde{\xi}_k) + E \quad (52a)$$

where

$$E = \frac{2\pi}{2^{2K} (2K)!} P_n^{(2K)}(\lambda)$$

and,

$$|\lambda| < 1.0$$

The error term  $E$  is zero if  $P_n$  is a polynomial of degree  $\leq 2K - 1$ .

A more accurate formula which utilizes the fact that  $P_n(1.0) = 0$  is used by Hsu (and by us)

$$f_0(\tilde{\xi}) = \sqrt{\frac{1 - \tilde{\xi}}{1 + \tilde{\xi}}}$$

$$f_n(\tilde{\xi}) = \sqrt{\frac{1 - \tilde{\xi}}{1 + \tilde{\xi}}} (1 + \tilde{\xi}) U_n(\tilde{\xi})$$

Then, the inner integral in Equation 51 may be written

$$I_1 = \int_{-1}^1 \sqrt{\frac{1-\tilde{\xi}}{1+\tilde{\xi}}} F_n(\tilde{\xi}) d\tilde{\xi} \quad (53)$$

where

$$F_0(\tilde{\xi}) = \bar{K}(x_j - \tilde{\xi}, \underline{s}_r, \underline{\sigma})$$

$$F_n(\tilde{\xi}) = (1 + \tilde{\xi}) U_n(\tilde{\xi}) \bar{K}(x_j - \tilde{\xi}, \underline{s}_r, \underline{\sigma}), \quad 1 \leq n$$

This is evaluated by the L-point Jacobi-Gauss quadrature with the weight function  $\sqrt{(1-\tilde{\xi})/(1+\tilde{\xi})}$  (see Reference 8, Chapter 8). The resulting formula

$$I \cong \sum_{k=1}^L W_k F_n(\tilde{\xi}_k) \quad (54)$$

is exact if  $F_n(\tilde{\xi})$  is a polynomial of degree  $\leq 2L-1$ , which corresponds to degree  $2L$  for  $P_n(\tilde{\xi})$ . Putting  $\tilde{\xi} = -\cos \theta$ , the polynomials

$$\phi_m(\tilde{\xi}) = \frac{\cos\left(m + \frac{1}{2}\right)\theta}{\cos\frac{1}{2}\theta}$$

are orthogonal with respect to the weight function  $\sqrt{(1-\tilde{\xi})/(1+\tilde{\xi})}$

$$\int_{-1}^1 \phi_m(\tilde{\xi}) \phi_n(\tilde{\xi}) \sqrt{\frac{1-\tilde{\xi}}{1+\tilde{\xi}}} d\tilde{\xi} = \begin{cases} 0, & m \neq n \\ \pi, & m = n \end{cases}$$

This is easily verified by expressing the integral in terms of  $\theta$ . Referring to formulas in Reference 8, Chapter 8, Section 8.4, it can be shown, using these polynomials, that Equation 54 takes the form

$$\int_{-1}^1 \sqrt{\frac{1-\tilde{\xi}}{1+\tilde{\xi}}} F_n(\tilde{\xi}) d\tilde{\xi} = \frac{2\pi}{2L+1} \sum_{k=1}^L H_k F_n(\tilde{\xi}_k) \quad (55)$$

where

$$H_k = (1 - \tilde{\xi}_k)$$

and

$$\tilde{\xi}_k = -\cos\left(\frac{2k-1}{2L+1} \pi\right)$$

In two-dimensional, steady, incompressible flow, there is an optimum set of chordwise collocation stations ( $\tilde{x}_j$ ) for the determination of sectional lift, depending upon the order of the polynomial required to adequately represent the downwash distribution

$$\tilde{x}_j = -\cos\left(\frac{2j}{2N+1} \pi\right), j = 1, 2, \dots, N. \quad (56)$$

Since the behavior of the integrand for the chordwise loading is apt to exhibit similar characteristics near the surface edges, it is inferred that this set should also yield the best approximation in three-dimensional, unsteady, compressible flow. Note that the number of collocation points is not required to be the same as the number of integration points. As will be seen later, it is only necessary that the total number of downwash collocation points be equal to or greater than the number of pressure coefficients  $a_{nm}$ . When  $N$  chordwise integration stations are used, the quadrature used to evaluate the inner integral of Equation 51 is exact for integrands represented by a polynomial of degree  $\leq 2N - 1$ .

Hsu shows that an optimum set of interdigitated spanwise collocation stations and integration stations exists for evaluation of the outer integral in Equation 51. By reasoning similar to that used to establish the chordwise collocation stations, it was established that the optimum spanwise collocation stations are

$$\frac{s}{-r} = -\cos \frac{r}{M+1} \pi, r = 1, 2, \dots, M. \quad (57)$$

It was observed that the quadrature for the integral of difference between the actual and polynomial approximation of the spanwise loading is zero when the actual loading is precisely represented by a polynomial of degree  $\leq 2M - 1$ , and the polynomial approximation is of degree  $= N - 1$ .

Then, by substitution of Equation 55 into Equation 51,

$$D_{nm}^i = \frac{1}{8\pi} \int_{-1}^1 \frac{\sqrt{1-\sigma^2}}{(\underline{s}_r - \sigma)^2} G_{nm}(\tilde{x}_j, \underline{s}_r, \sigma) d\sigma \quad (58)$$

where

$$G_{om} = \frac{2\pi}{2L+1} \sum_{k=1}^L \sigma^m (1 - \tilde{\xi}_k) (\underline{s}_r - \sigma)^2 \overline{K}(\underline{x}_j - \xi_k, \underline{s}_r, \sigma) \quad (59)$$

$$G_{nm} = \frac{2\pi}{2L+1} \sum_{k=1}^L \sigma^m (1 - \tilde{\xi}_k^2) U_n(\tilde{\xi}_k) (\underline{s}_r - \sigma)^2 \overline{K}(\underline{x}_j - \xi_k, \underline{s}_r, \sigma); 1 \leq n$$

Hsu established the form of the Gaussian quadrature and the spanwise integration stations. The difficulties of the singularity of the kernel at  $\sigma = \underline{s}_r$  and the difficulty of differentiation with respect to  $\sigma$  (he uses the steady-state lifting line formula to derive the form of the quadrature) are avoided by removal of the singularity at  $\sigma = \underline{s}_r$  and then by an integration by parts.

We first integrate by parts to get

$$D_{nm}^i = \frac{1}{8\pi} \int_{-1}^1 \frac{\frac{\partial}{\partial \sigma} \left[ \sqrt{1-\sigma^2} G_{nm}(\tilde{x}_j, \underline{s}_r - \sigma, \omega, M) \right]}{(\underline{s}_r - \sigma)} d\sigma$$

which corresponds to Equation 58 in Reference 9. The Gaussian quadrature formula is developed and shows that when the number of integration stations is one greater than the number of collocation stations, and if they are interdigitated in the prescribed way

$$D_{nm}^i = \frac{1}{8\pi} \left\{ \sum_{p=1}^{M+1} \frac{\pi}{M+1} \frac{(1 - \sigma_p^2) G_{nm}(\tilde{x}_j, \underline{s}_r, \underline{\sigma}_p)}{(\underline{s}_r - \underline{\sigma}_p)^2} - \pi(M+1) G_{nm}(\tilde{x}_j, \underline{s}_r, \underline{s}_r) \right\} \quad (60)$$

where

$$\underline{s}_r = -\cos \frac{r\pi}{M+1}$$

and

$$\underline{\sigma}_p = -\cos \frac{2p-1}{2(M+1)} \pi$$

$$r = 1, 2, \dots, M$$

Evaluation of the second term in the brackets requires the observation that the multiplier of  $K_2$  in Equation 48 goes to zero whenever the collocation point is in the plane of the doublet sheet located at the integration point. Therefore, the finite part of the integral of the  $K_2$  term is zero, and the entire contribution comes from the  $K_1$  term. In this case,  $\gamma(\underline{s}) = \gamma(\underline{\sigma})$  and  $r_1^2 = (\underline{s}_r - \underline{\sigma})^2 = 0$ .

It can be shown that

$$K_1(x - \xi, \underline{s}_r, \underline{s}_r) = \begin{cases} -2, & x > \xi \\ 0, & x < \xi. \end{cases}$$

Thus, the chordwise integral which defines  $G_{nm}(x_j, \underline{s}_r, \underline{s}_r)$  is

$$G_{nm}(x_j, \underline{s}_r, \underline{s}_r) = -2 \int_{-1}^{\tilde{x}_j} \underline{\sigma}^m f_n(\tilde{\xi}) e^{-i \frac{\omega}{U}(x_j - \xi)} d\tilde{\xi}$$

If the range of integration is extended to  $\tilde{\xi} = 1$  by making the integrand zero for  $\tilde{\xi} > \tilde{x}_j$ , the integral cannot be well approximated by a polynomial because it has a jump discontinuity at  $\tilde{\xi} = \tilde{x}_j$ . To overcome this difficulty, we write

$$G_{nm}(x_j, \underline{s}_r, \underline{s}_r) = -2 \int_{-1}^{\tilde{x}_j} \underline{\sigma}^m f_n(\tilde{\xi}) \left[ e^{-i \frac{\omega}{U} (x_j - \tilde{\xi})} - 1 \right] d\tilde{\xi} \quad (61a)$$

$$-2 \int_{-1}^{\tilde{x}_j} \underline{\sigma}^m f_n(\tilde{\xi}) d\tilde{\xi}.$$

The second term here depends on the integrals

$$h_n(\tilde{x}_j) = \int_{-1}^{\tilde{x}_j} f_n(\tilde{\xi}) d\tilde{\xi}$$

which may be evaluated exactly. We have

$$h_0(\tilde{x}_j) = \int_{-1}^{\tilde{x}_j} \sqrt{\frac{1-\tilde{\xi}}{1+\tilde{\xi}}} d\tilde{\xi} = \frac{\pi}{2} + \sin^{-1} \tilde{x}_j + \sqrt{1-\tilde{x}_j^2}$$

$$h_1(\tilde{x}_j) = \int_{-1}^{\tilde{x}_j} \sqrt{1-\tilde{\xi}^2} d\tilde{\xi} = \frac{1}{2} \left( \frac{\pi}{2} + \sin^{-1} \tilde{x}_j + \tilde{x}_j \sqrt{1-\tilde{x}_j^2} \right)$$

This list may be extended as far as it is needed.

The first integral in Equation 61a is considered as an integral over  $-1 < \tilde{\xi} < 1$  with zero integrand when  $\tilde{\xi} > \tilde{x}_j$ , and Equation 55 is applied. The result is

$$G_{om}(x_j, \underline{s}_r, \underline{s}_r) = -2 \cdot \frac{2\pi}{2L+1} \sum_{\substack{k \geq 1, \\ \tilde{\xi}_k < \tilde{x}_j}} \underline{\sigma}^m (1 - \tilde{\xi}_k) \left[ e^{-i \frac{\omega}{U} (x_j - \tilde{\xi}_k)} - 1 \right] \\ - 2 \underline{\sigma}^m g_o(\tilde{x}_j) \quad (61b)$$

$$G_{nm}(x_j, s_r, s_r) = -2 \frac{2\pi}{2L+1} \sum_{\substack{k \geq 1, \\ \tilde{\xi}_k < \tilde{x}_j}} \sigma^m (1 - \tilde{\xi}_k^2) U_n(\tilde{\xi}_k) \left[ e^{-i \frac{\omega}{U} (x_j - \xi_k)} - 1 \right]$$

$$- 2\sigma^m g_n(\tilde{x}_j), \quad n \geq 1.$$

Equations 59 and 61 are used to evaluate the  $D_{nm}^i$ 's given by Equation 60. Equation 50 is then used to determine the pressure coefficients  $a_{nm}$  to match the downwashes (i.e. the  $\bar{w}_i/U$ 's) at the collocation points  $(x_j, s_r)$ . Once the pressure coefficients are determined, the generalized forces are computed. A polynomial expression for the  $i^{\text{th}}$  modal deflections normal to the surface,

$$n^{(i)}(\xi, \sigma) = \sum_{\nu=0}^N \sum_{\mu=0}^M b_{\nu\mu}^{(i)} \xi^{\nu} \sigma^{\mu}$$

and Equation 49 are substituted into the equation

$$Q_{ij} = \int_{-s_{\text{TIP}}}^{s_{\text{TIP}}} \int_{x_{\text{LE}}}^{x_{\text{TE}}} n^{(i)}(\xi, \sigma) \Delta \bar{p}^{(j)}(\tilde{\xi}, \sigma) d\tilde{\xi} d\sigma$$

to get

$$Q_{ij} = s_{\text{TIP}}^2 (1/2 \rho U^2) \sum_{n=0}^N \sum_{m=0}^M \sum_{\nu=0}^N \sum_{\mu=0}^M a_{nm}^{(j)} b_{\nu\mu}^{(i)} \Delta Q_{nm\nu\mu} \quad (62)$$

where

$$\Delta Q_{nm\nu\mu} = \int_{-1}^1 \int_{-1}^1 \sqrt{1 - \sigma^2} \sigma^{m+u} \xi^{\nu} \sqrt{\frac{1 - \tilde{\xi}}{1 + \tilde{\xi}}} F_n(\tilde{\xi}) d\tilde{\xi} d\sigma$$

The quadrature formula given by Equation 55 with  $L$  set equal to  $N$  may be used to evaluate the inner integral. For evaluation of the outer integral, a quadrature formula with the weight function  $\sqrt{1 - \sigma^2}$ , analogous to Equation 60 for a nonsingular integral, may be used. This is not practical for a wing

with folded tip, for which a different representation should be used on each plane part of the surface. Here, the spanwise integral over one part of the wing may have a square root factor at one end of the interval of integration, or at neither end. Such integrals may best be evaluated by a suitable application of ordinary Gaussian quadrature with a weight function of 1.0. The points and weights for this quadrature method are not given by simple formulas such as Equation 55. They are listed in many places, (e. g. Reference 5).

## VI. APPLICATION TO A WING WITH A FOLDED TIP

The planform may be any continuous surface. However, the computer program was developed to treat either a planar or nonplanar planform of the type shown in Figure 2. (Only one-half the planform is shown.) To facilitate modifications of the program, comment cards are placed throughout the program to indicate where changes may be made to handle other nonplanar surfaces like that of the Paraglider.

In the application of the kernel function method to the planform shown in Figure 2, the computer program calculates from the equations of the leading and trailing edges the collocation and integration points for which the integrands of Equation 51 must be evaluated. For demonstration purposes, we calculate the collocation and integration points for values of  $L = 4$ ,  $N = 6$ , and  $M = 10$  and show the results in Figure 3.

$$x_{LE} = s \tan \lambda_{LE}, \quad x_{LETIP} = s_{FL} \tan \lambda_{LE} + (s - s_{FL}) \tan \lambda_{LETIP}$$

$$x_{TE} = 2b_o + s \tan \lambda_{TE}, \quad x_{TETIP} = 2b_o + s_{FL} \tan \lambda_{TE} + (s - s_{FL}) \tan \lambda_{TETIP}$$

$$b(s) = 1/2 (x_{TE} - x_{LE})$$

$$x_m = 1/2 (x_{LE} + x_{TE})$$

$$x = x_m + b(s) \tilde{x}$$

$$\xi = \xi_m + b(\sigma) \tilde{\xi}$$

$$\tilde{x}_j = -\cos \frac{2j}{2N+1} \pi, \quad j = 1, 2, \dots, N.$$

$$\tilde{\xi}_k = -\cos \frac{2k-1}{2L+1} \pi, \quad k = 1, 2, \dots, L.$$

$$s = \underline{s} s_{TIP}$$

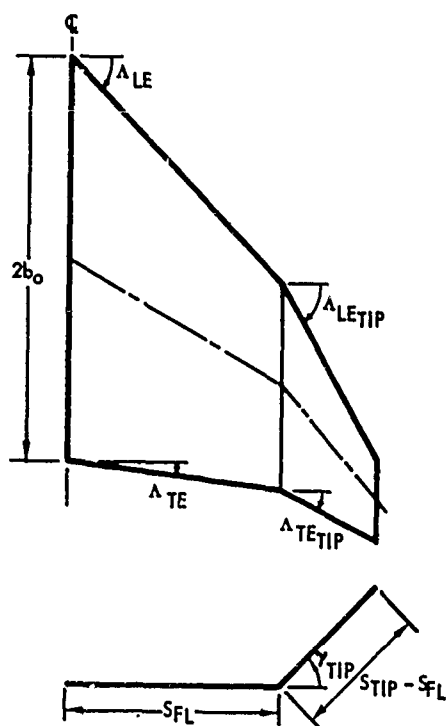


Figure 2. Planar Wing With Planar Symmetrically Folded Tips

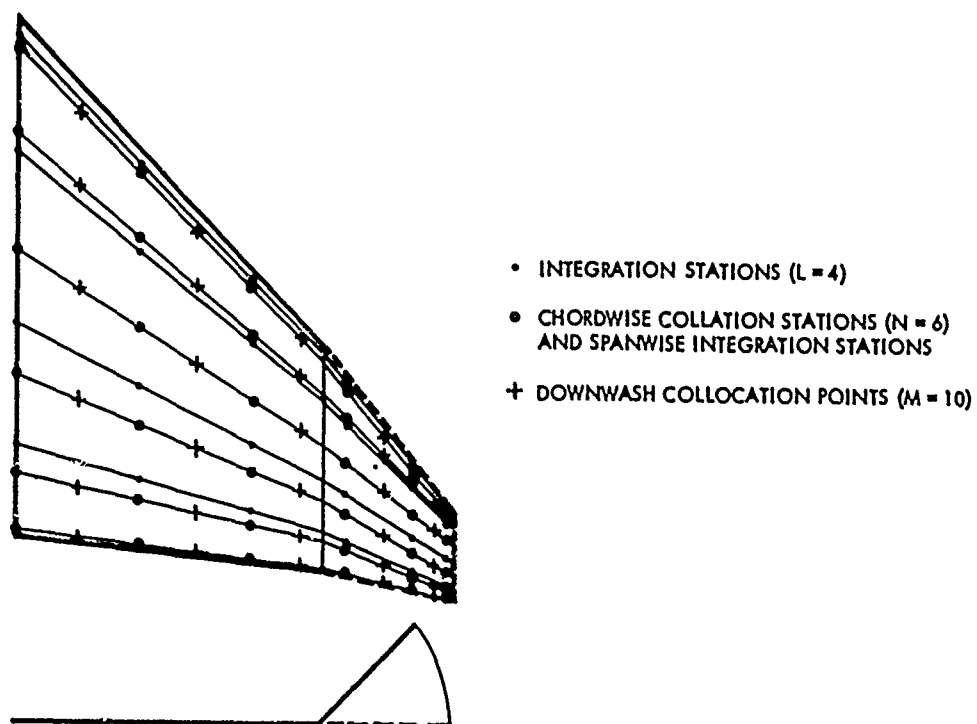


Figure 3. An Optimum Set of Collocation and Integration Points

$$\underline{s}_m = -\cos \frac{m\pi}{M+1}, \quad m = 1, 2, \dots, M.$$

$$\underline{\sigma}_p = -\cos \frac{2p-1}{2(M+1)} \pi, \quad p = 1, 2, \dots, (M+1).$$

We have also constructed a table of equations (Table 1), for the functions  $y_0$ ,  $z_0$ ,  $r_1$ ,  $T_1$ , and  $T_2$  for use in the expression for the kernel, Equations 48 through 48c. In Table 1, the subscript  $F_L$  indicates the fold line.

A difficulty is encountered in the spanwise integration because the kernel function has a finite discontinuity at the fold line. For example, note in Table 1 the change in  $T_1$  and  $T_2$  for receiving or collocation points on the wing as the sending or integration points shift from the port tip to the wing and from the wing to the starboard tip.

If we consider the kernel as a function of  $\tilde{\xi}$  and  $\underline{\sigma}$  for fixed values of  $x$  and  $\underline{s}$ ,

$$q(\tilde{\xi}, \underline{\sigma}) = \bar{K}(x - \xi, \underline{s}, \underline{\sigma}),$$

this function may be broken up into a simple discontinuous part  $g^{**}$  and a part  $g^*$  which is continuous across the fold lines:

$$g(\tilde{\xi}, \underline{\sigma}) = g^*(\tilde{\xi}, \underline{\sigma}) + g^{**}(\tilde{\xi}, \underline{\sigma}) \quad (63)$$

To do this, define

$$g^{**}(\tilde{\xi}, \underline{\sigma}) = \begin{cases} g(\tilde{\xi}, \underline{\sigma}_{F_L}^+) - g(\tilde{\xi}, \underline{\sigma}_{F_L}^-), & \underline{\sigma} > \underline{\sigma}_{F_L} \\ 0, & -\underline{\sigma}_{F_L} < \underline{\sigma} < \underline{\sigma}_{F_L} \\ g(\tilde{\xi}, -\underline{\sigma}_{F_L}^-) - g(\tilde{\xi}, -\underline{\sigma}_{F_L}^+), & \underline{\sigma} < -\underline{\sigma}_{F_L} \end{cases} \quad (64)$$

and then define  $g^*$  by Equation 63.

More explicitly, for  $\underline{\sigma} > \underline{\sigma}_{F_L}$ , define

$$\xi_{F_L} = b(\underline{\sigma}_{F_L}) \tilde{\xi} + \frac{1}{2} \left[ \xi_{LE}(\underline{\sigma}_{F_L}) + \xi_{TE}(\underline{\sigma}_{F_L}) \right]$$

Table 1. Spanwise Parameter in the Kernel Function

| Receiving Points (x, s)   |                                  |                           |   |  |                      |                                 |   |                           |                                 |
|---|----------------------------------|---------------------------|---|--|----------------------|---------------------------------|---|---------------------------|---------------------------------|
| Port Tip  |                                  |                           |   | Wing                                       |                      |                                 | Starboard Tip   |                           |                                 |
| $S < -Y_{FL}$ and $\gamma(S) = -\gamma_{TIP}$   |                                  |                           |   | $-Y_{FL} < S < Y_{FL}$ and $\gamma(S) = 0$ |                      |                                 | $Y_{FL} < S$ and $\gamma(S) = \gamma_{TIP}$                                   |                           |                                 |
| Sending Points ( $\xi, \sigma$ )  |                                  |                           |   |  |                      |                                 |   |                           |                                 |
| Functions   | PT                               | W                         | ST  | PT   | W                    | ST                              | PT  | W                         | ST                              |
|   | $\gamma(\sigma) = -\gamma_{TIP}$ | $\gamma(\sigma) = 0$      | $\gamma(\sigma) = \gamma_{TIP}$   | $\gamma(\sigma) = \gamma_{TIP}$            | $\gamma(\sigma) = 0$ | $\gamma(\sigma) = \gamma_{TIP}$ | $\gamma(\sigma) = -\gamma_{TIP}$  | $\gamma(\sigma) = 0$      | $\gamma(\sigma) = \gamma_{TIP}$ |
| $Y_0$   | $a_0 \xi$                        | $-\sigma^2 + a_0^2 \xi$   | $(a_0^2 - \sigma_0^2) \xi - 2\gamma_{FL}$   | $a_0^2 - \sigma^2 \xi$                     | $a_0$                | $a_0 - \sigma_0 \xi$            | $(a_0 - \sigma^2) \xi + 2\gamma_{FL}$   | $-\sigma_0 + a_0 \xi$     | $a_0 \xi$                       |
| $z_0$   | $-a_0 \beta$                     | $-a_0^2 \beta$            | $-(a_0 + \sigma) \beta$   | $\sigma^2 \beta$                           | 0                    | $-\sigma_0 \beta$               | $(a_0 + \sigma^2) \beta$  | $a_0 \beta$               | $a_0 \beta$                     |
| $r_1$   | $ a_0 $                          | $\sqrt{a_0^2 + \sigma^2}$ | $\sqrt{[(a_0^2 - \sigma_0^2) \xi - 2\gamma_{FL}]^2 + [(a_0^2 + \sigma_0^2) \beta]^2}$ | $\sqrt{a_0^2 + \sigma^2}$                  | $ a_0 $              | $\sqrt{a_0^2 + \sigma^2}$       | $\sqrt{[(a_0 - \sigma^2) \xi + 2\gamma_{FL}]^2 + [(a_0 + \sigma^2) \beta]^2}$ | $\sqrt{a_0^2 + \sigma^2}$ | $ a_0 $                         |
| $T_1$   | 1.0                              | $\xi$                     | $\xi^2 - \beta^2$   | $\xi$                                      | 1.0                  | $\xi$                           | $\xi^2 - \beta^2$   | $\xi$                     | 1.0                             |
| $T_2$   | 0                                | $a_0^2 \beta^2$           | $4(a_0 \xi + \gamma_{FL})(a_0^2 \xi - \gamma_{FL}) \beta^2$                           | $a_0^2 \beta^2$                            | 0                    | $a_0^2 \beta^2$                 | $4(a_0^2 \xi - \gamma_{FL})(a_0 \xi + \gamma_{FL}) \beta^2$                   | $a_0^2 \beta^2$           | 0                               |
| $a_0^2 = a_0 + \gamma_{FL}$ , $\sigma^2 = \sigma + \gamma_{FL}$ , $a_0 = a - \gamma_{FL}$ , $\sigma_0 = \sigma - \gamma_{FL}$ , $a = a \sin \gamma_{TIP}$ , $\beta = \cos \gamma_{TIP}$ |                                  |                           |   |  |                      |                                 |   |                           |                                 |

Then

$$g^{**}(\tilde{\xi}, \underline{\sigma}) = \bar{K}^{ST} (x - \xi_{FL}, \underline{\sigma}, \underline{\sigma}_{FL}) - \bar{K}^W (x - \xi_{FL}, \underline{\sigma}, \underline{\sigma}_{FL})$$

in which

$$v(\underline{\sigma}) = v_{TIP} \text{ in } \bar{K}^{ST} \quad (ST \sim \text{starboard tip})$$

$$v(\underline{\sigma}) = 0 \text{ in } \bar{K}^W \quad (W \sim \text{wing})$$

A similar formula applies when  $\underline{\sigma} < -\underline{\sigma}_{FL}$ .  $g^{**}$  may be written in a form which indicates that it is independent of  $\sigma$  in each tip region:

$$g^{**}(\tilde{\xi}, \underline{\sigma}) = \begin{cases} g_{ST}^{**}(\tilde{\xi}), & \underline{\sigma} > \underline{\sigma}_{FL} \\ 0, & -\underline{\sigma}_{FL} < \underline{\sigma} < \underline{\sigma}_{FL} \\ g_{PT}^{**}(\tilde{\xi}), & \underline{\sigma} < -\underline{\sigma}_{FL} \end{cases} \quad (65)$$

With the use of Equations 63 and 65, Equation 51 may be rewritten as

$$\begin{aligned} D_{mn}^i &= \frac{1}{8\pi} \int_{-1}^1 \sqrt{1 - \underline{\sigma}^2} \underline{\sigma}^m \int_{-1}^1 f_n(\tilde{\xi}) g^*(\tilde{\xi}, \underline{\sigma}) d\tilde{\xi} d\underline{\sigma} \\ &+ \frac{1}{8\pi} \int_{-1}^1 f_n(\tilde{\xi}) g_{ST}^{**}(\tilde{\xi}) \int_{\underline{\sigma}_{FL}}^1 \sqrt{1 - \underline{\sigma}^2} \underline{\sigma}^m d\underline{\sigma} d\tilde{\xi} \\ &+ \frac{1}{8\pi} \int_{-1}^1 f_n(\tilde{\xi}) g_{PT}^{**}(\tilde{\xi}) \int_{-1}^{-\underline{\sigma}_{FL}} \sqrt{1 - \underline{\sigma}^2} \underline{\sigma}^m d\underline{\sigma} d\tilde{\xi} \end{aligned}$$

The first of these three double integrals is calculated according to Equation 60. In the others, the inner integral may be evaluated exactly. The constants

$$u_m = \int_{\frac{\sigma}{F_L}}^1 \sqrt{1 - \sigma^2} \sigma^m d\sigma$$

are given by formulas

$$u_0 = \frac{1}{2} \left[ \frac{\pi}{2} - \sin^{-1} \frac{\sigma}{F_L} + \frac{\sigma}{F_L} \sqrt{1 - \frac{\sigma^2}{F_L^2}} \right]$$

$$u_1 = \frac{1}{3} (1 - \frac{\sigma^2}{F_L^2})^{3/2}$$

etc.

In terms of these constants

$$D_{mn}^i = \frac{1}{8\pi} \int_{-1}^1 \sqrt{1 - \sigma^2} \sigma^m \int_{-1}^1 f_n(\tilde{\xi}) g^*(\tilde{\xi}, \sigma) d\tilde{\xi} d\sigma$$

$$+ \frac{1}{8\pi} u_m \int_{-1}^1 f_n(\tilde{\xi}) \left[ g_{ST}^{**}(\tilde{\xi}) + (-1)^m g_{PT}^{**}(\tilde{\xi}) \right] d\tilde{\xi}$$

The last integral in this formula is evaluated by Equation 55.

In the evaluation of  $O_{ij}$ , given by Equation 62 for the plane case, it is assumed that modes numbers  $i$  and  $j$  are either both symmetric in  $\sigma$ , or both antisymmetric. Then the contribution to the integral for  $\sigma < 0$  is the same as the contribution for  $\sigma > 0$ . Let the deflection in the  $i^{\text{th}}$  mode be given by

$$n^{(i)}(\xi, \sigma) = \begin{cases} \sum_{\nu=0}^N \sum_{\mu=0}^M c_{\nu\mu}^{(i)} \xi^{\nu} \sigma^{\mu}, & 0 < \sigma < \sigma_{FL} \\ \sum_{\nu=0}^N \sum_{\mu=0}^M d_{\nu\mu}^{(i)} \xi^{\nu} \sigma^{\mu}, & \sigma > \sigma_{FL} \end{cases}$$

Then

$$O_{ij} = 2 \cdot y_{TIP}^2 \left( \frac{1}{2} \rho U^2 \right) \sum_{n=0}^N \sum_{m=0}^M \sum_{\nu=0}^N \sum_{\mu=0}^M a_{nm}^{(j)} \left\{ c_{\nu\mu}^{(i)} I_{nm\nu\mu}^{(1)} + d_{\nu\mu}^{(i)} I_{nm\nu\mu}^{(2)} \right\}$$

in which

$$I_{nm\nu\mu}^{(1)} = \int_0^{\sigma_{FL}} \sqrt{1 - \sigma^2} \sigma^{m+\mu} \int_{-1}^1 \xi^\nu \sqrt{\frac{1 - \tilde{\xi}}{1 + \tilde{\xi}}} F_n(\tilde{\xi}) d\tilde{\xi} d\sigma$$

$$I_{nm\nu\mu}^{(2)} = \int_{\sigma_{FL}}^1 \sqrt{1 - \sigma^2} \sigma^{m+\mu} \int_{-1}^1 \xi^\nu \sqrt{\frac{1 - \tilde{\xi}}{1 + \tilde{\xi}}} F_n(\tilde{\xi}) d\tilde{\xi} d\sigma$$

Note that the inner integrals are the integrals of polynomials in  $\tilde{\xi}$  multiplied by  $\sqrt{(1 - \tilde{\xi})/(1 + \tilde{\xi})}$ , by virtue of the relation

$$\xi = b(\sigma)\tilde{\xi} + \frac{1}{2} \left[ \xi_{LE}(\sigma) + \xi_{TE}(\sigma) \right]$$

Hence, the inner integral may be evaluated exactly by either Equation 52a or Equation 55 if enough points are used in the formulas. For the limits  $\nu \leq 5$ ,  $n \leq 4$  used in the computer program, six points are sufficient for Equation 52a, five points for Equation 55. Equation 52a was used with  $K = 6$ . (This choice was made arbitrarily; it would be just as good to use Equation 55.)

In the integrations over  $\sigma$ , six-point Gaussian integration with weight function 1.0 was used. The basic formula is

$$\int_0^1 f(v) dv = \sum_{l=1}^6 h_l f(v_l) \quad (66)$$

exact for  $f(v)$  a polynomial of degree at most 11. The constants occurring in this formula are given in the subroutine FORCE. They may be derived from a table given by Scarborough (Reference 6, p. 148).

In  $I_{nm\nu\mu}^{(1)}$ , Equation 66 is applied by putting

$$\underline{\sigma} = \underline{\sigma}_{F_L} v$$

The resulting expression is

$$I_{nm\nu\mu}^{(1)} = \frac{\pi}{6} \underline{\sigma}_{F_L} \sum_{\ell=1}^6 h_{\ell} \sqrt{1 - \underline{\sigma}_{\ell}^2} \underline{\sigma}_{\ell}^{m+\mu} \sum_{k=1}^6 \xi_k(\underline{\sigma}_{\ell})^{\nu} (1 - \tilde{\xi}_k^2)^{F_n(\tilde{\xi}_k)}$$

in which

$$\underline{\sigma}_{\ell} = \underline{\sigma}_{F_L} v_{\ell}$$

$$\xi_k(\underline{\sigma}_{\ell}) = b(\underline{\sigma}_{\ell}) \tilde{\xi}_k + \frac{1}{2} \left[ \xi_{LE}(\underline{\sigma}_{\ell}) + \xi_{TE}(\underline{\sigma}_{\ell}) \right]$$

In  $I_{nm\nu\mu}^{(2)}$ , the transformation

$$\underline{\sigma} = 1 - (1 - \underline{\sigma}_{F_L}) v^2$$

was used. This makes the  $v$ -integrand behave like a polynomial at the ends of the interval. The resulting formula is

$$I_{nm\nu\mu}^{(2)} = \frac{\pi}{3} (1 - \underline{\sigma}_{F_L}) \sum_{\ell=1}^6 h_{\ell} v_{\ell} \sqrt{1 - \underline{\sigma}_{\ell}^2} \underline{\sigma}_{\ell}^{m+\mu} \sum_{k=1}^6 \xi_k(\underline{\sigma}_{\ell})^{\nu} (1 - \tilde{\xi}_k^2)^{F_n(\tilde{\xi}_k)}$$

in which

$$\underline{\sigma}_{\ell} = 1 - (1 - \underline{\sigma}_{F_L}) v_{\ell}^2$$

$$\xi_k(\underline{\sigma}_{\ell}) = b(\underline{\sigma}_{\ell}) \tilde{\xi}_k + \frac{1}{2} \left[ \xi_{LE}(\underline{\sigma}_{\ell}) + \xi_{TE}(\underline{\sigma}_{\ell}) \right]$$

## DESCRIPTION OF THE COMPUTER PROGRAM

A functional diagram of the computer program is given in Figure 4. With the exception of two subroutines named MSIMEC and MSIMER, all of the programs are written in Fortran IV. These two subroutines, written in machine language, are used for complex and real matrix inversion, respectively. There are certain limitations related to the various other subprograms, which are listed below.

| Subprogram      | Limitations   |
|-----------------|---|
| Subroutine Data |   |
| NCC             | The number of chordwise collocation stations must be $\leq 10$ .  |
| NCS             | The number of spanwise collocation stations must be $\leq 9$ .  |
| NDATA           | The number of sets of data must be $\leq 10$ .  |
| N               | The number of chordwise pressure modes $\leq 5$ .   |
| M               | The number of spanwise pressure modes $\leq 5$ .  |
| Subroutine Zen  |   |
| MODES           | This is the number of modes used in the calculation of generalized forces, and must be $\leq 10$ .  |
| NPTS            | For a planar wing, this is the number of points at which the deflection is given in the horizontal surface and must be $\leq 66$ . For a nonplanar wing, there must be $\leq 66$ points for the deflections in the horizontal surface and $\leq 66$ points for the deflections in the vertical surface. |

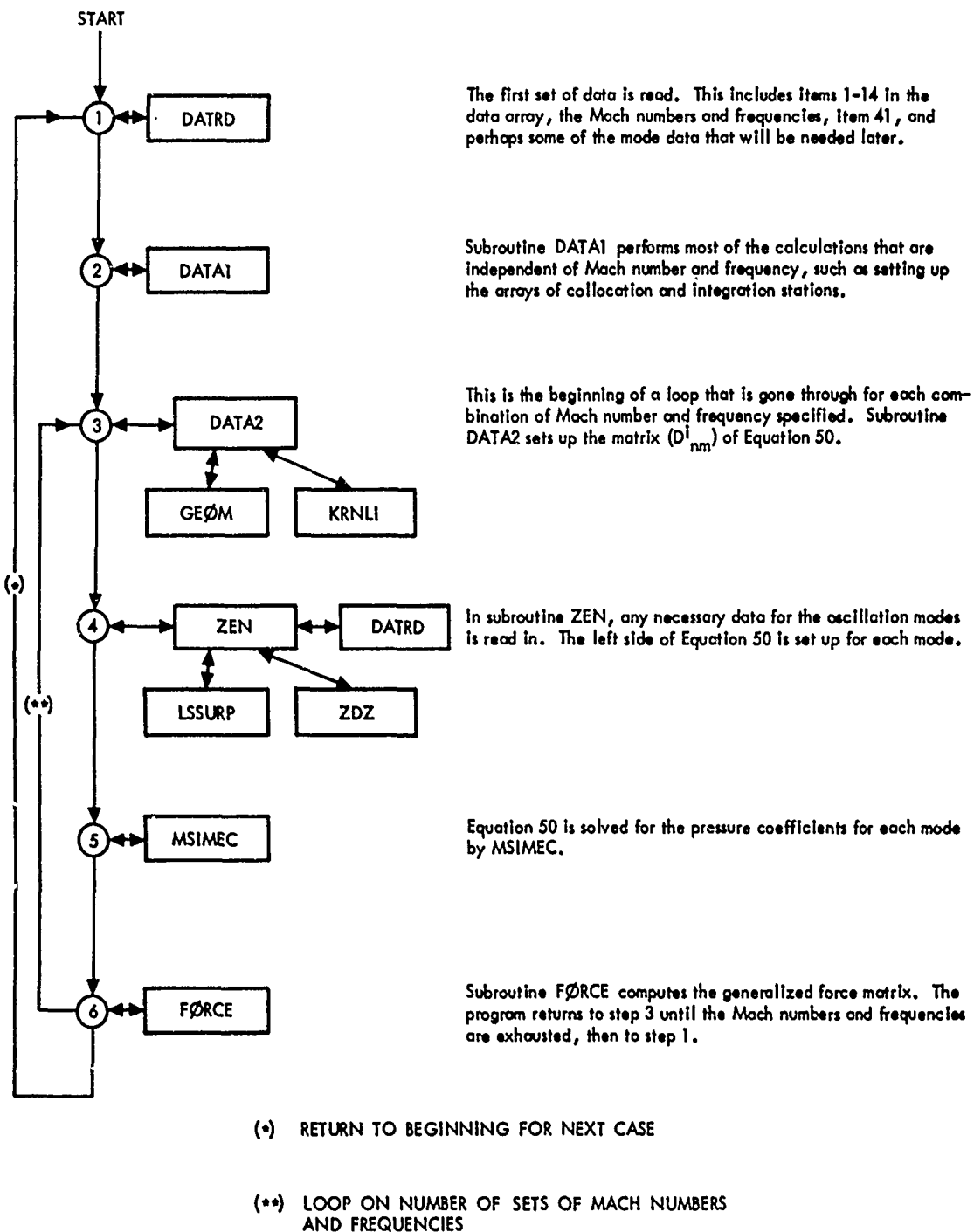


Figure 4. Functional Flow Diagram—Main Program

## USE OF THE COMPUTER PROGRAM

The following rules apply to optimum use of the computer program:

1. Before attempting to use the computer program compute the coefficients of polynomials of minimum order in  $x$  and  $y$  that will adequately represent the modal deflection distributions. Least-square fitted surfaces are very useful for this purpose, since weighting factors may be used to obtain a better fit in special regions.
2. Set the number of chordwise collocation points  $M$  equal to one plus the highest of the orders in  $x$ ; and, unless there is special reason to reduce the number of chordwise integration points, set  $L$  equal to  $M$ .
3. Set the number of spanwise collocation points  $R$  equal to one plus the highest of the orders in  $y$ . This establishes the number of pressure coefficients  $a_{nm}$  at  $M \times R$ . Their values are computed by matching exactly the downwashes at the  $M \times R$  spanwise collocation points.

The input data is read by the subroutine DATRD. Use of this subroutine requires that, on each data card, the first 72 columns are six fields of width 12, as indicated on the sample data sheets (Figure 5). The first field contains an integer giving the location in the data array in which the number in the second field is to be stored. The numbers in the remaining fields are stored in consecutive locations. If a field is blank, the corresponding location in the data array is unchanged. DATRD reads any number of cards. A minus sign in column 1 indicates the last card to be read; if this minus sign is not present, DATRD continues with the next card. The storage locations of the data on a card are not affected by the sign in column 1. All floating point numbers must be written with decimal points. All integers must be at the right of their fields.

The data array is set up as follows:

- |          |  |
|----------|--|
| 1. N     | The number of chordwise pressure modes   |
| 2. M     | The number of spanwise pressure modes  |
| 3. NCC   | The number of chordwise collocation points   |
| 4. NCS   | The number of spanwise collocation points  |
| 5. NDATA | The number of sets of values of Mach number and frequency to be used.                  |
| 6. NSYM  | Indicator for symmetric (NSYM = +1) or antisymmetric (NSYM = -1) modes of oscillation. |

- |        |   |   |
|--------|---|---|
| 7.     | SFOLD   | Distance spanwise from wing center line to fold line  |
| 8.     | S'TIP   | Semispan  |
| 9.     | BO  | One-half of the root chord  |
| 10.    | ALFA1   | The fold angle (in degrees)   |
| 11.    | $\lambda_{LE}$  | The sweep angle of the leading edge (in degrees)  |
| 12.    | $\lambda_{TE}$  |   |
| 13.    | $\lambda_{LETIP}$                                     |   |
| 14.    | $\lambda_{TETIP}$                                     |   |
| 21-30. | Values of Mach number.                                | NDATA of these must be entered.   |
| 31-40. | Values of reduced frequency,<br>$\omega \cdot BO/U$ . | NDATA of these must be entered.   |
| 41.    | NMOD  | The number of modes   |
| 42.    | JD  | Indicator for the type of input data for a mode. If JD=1, the deflections are given at a set of points on the wing (and tip). If JD=2, the coefficients of polynomials for the deflection of the wing and tip are given. If JD=0, the current mode and subsequent modes are not given by data. They are the same as the corresponding modes which were used for the previous frequency and Mach number. |
| 43.    | NPTSW   | Number of points on the wing at which deflections are given (used if JD=1).   |
| 44.    | NPTST   | Number of points on tip at which deflections are given.   |
| 51-71. | Deflection coefficients on the wing.                  | The coefficients are stored as follows:   |

$$a_{00} + a_{10}x + a_{20}x^2 + a_{30}x^3 + a_{40}x^4 + a_{50}x^5 + y(a_{01} + a_{11}x + a_{21}x^2$$

$$+ a_{31}x^3 + a_{41}x^4) + y^2(a_{02} + a_{12}x + a_{22}x^2 + a_{32}x^3)$$

$$+ y^3(a_{03} + a_{13}x + a_{23}x^2) + y^4(a_{04} + a_{14}x) + a_{05}y^5$$

- 76-96. Deflection coefficients on the tip. Same storage rule as above, except all locations are increased by 25.
98. Indicator that no more modes are to be read after the present one.  
(For current and subsequent frequencies and Mach numbers.)
- 101-299. Deflection data at points on the wing, in the order  $x_1, s_1, n_1, x_2, s_2, n_2$ , etc.
- 301-499. Deflection data at points on the tip.

Items 1-41 must be read in the first set of data. There may be an additional set of data for each mode, for each Mach number and frequency case. After the indicator DAT(98) has been given a non-zero value, no more deflection data will be read. After JD has been given the value zero, no data will be read for the higher numbered modes.

The data in Figure 5 is for a  $60^\circ$  triangular wing folded at 75 percent semispan, at an angle of  $30^\circ$ . The root chord is 5.0 feet, making  $BO = 2.5$ . Three modes: plunge, pitch, and a third nonrigid mode are considered. The Mach number is 0.7, and six frequencies: 10, 20, 30, 40, 50, and 60 cps are used. The speed of sound is taken to be 1000 ft./sec. This gives reduced frequencies of 0.157, 0.314, 0.471, 0.628, 0.785, and 0.942.

Three spanwise and three chordwise pressure modes, six chordwise and eight spanwise collocation stations are specified.

In the set of cards numbered 15 - 30, which give deflection data, some of the cards have been omitted. Otherwise, this is a complete set of data for a computer run.

# FORTRAN FIXED IO DIGIT DECIMAL DATA

DECK NO. \_\_\_\_\_ PROGRAMMER \_\_\_\_\_ DATE \_\_\_\_\_ PAGE 1 of 5 JOB NO. \_\_\_\_\_

| NUMBER | IDENTIFICATION | DESCRIPTION | DO NOT KEY PUNCH |
|--------|----------------|-------------|------------------|
| 1      | 1              |             |                  |
| 13     |                | N           |                  |
| 25     |                | H           |                  |
| 37     |                | NCC         |                  |
| 49     |                | NCS         |                  |
| 61     |                | NDATA       |                  |
| 1      | 2              |             |                  |
| 13     |                | NSYM        |                  |
| 25     |                | SEOLD       |                  |
| 37     |                | STIP        |                  |
| 49     |                | BO          |                  |
| 61     |                | Fold Angle  |                  |
| 1      | 3              |             |                  |
| 13     |                |             |                  |
| 25     |                |             |                  |
| 37     |                |             |                  |
| 49     |                |             |                  |
| 61     |                |             |                  |
| 1      | 4              |             |                  |
| 13     |                |             |                  |
| 25     |                |             |                  |
| 37     |                |             |                  |
| 49     |                |             |                  |
| 61     |                |             |                  |

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Figure 5. Sample Data Sheets (Sheet 1 of 5)

# FORTRAN FIXED IO DIGIT DECIMAL DATA

| DECK NO. _____                 |         | PROGRAMMER _____ |  | DATE _____  |  | PAGE 2 of 5      |  | JOB NO. _____ |  |
|--------------------------------|---------|------------------|--|---|--|------------------|--|---------------|--|
| NUMBER                         |         | IDENTIFICATION   |  | DESCRIPTION   |  | DO NOT KEY PUNCH |  |               |  |
| 1                              | 2 6     | [Shaded Box]     |  | Last Mach Number                                    |  |                  |  |               |  |
| 13                             | 0 7     |                  |  |   |  |                  |  |               |  |
| 25                             |         |                  |  |   |  |                  |  |               |  |
| 37                             |         |                  |  |   |  |                  |  |               |  |
| 49                             |         |                  |  |   |  |                  |  |               |  |
| 61                             |         | 73 80            |  |   |  |                  |  |               |  |
| 1                              | 3 1     | [Shaded Box]     |  | Reduced Frequencies                                 |  |                  |  |               |  |
| 13                             | 0 1 5 7 |                  |  |   |  |                  |  |               |  |
| 25                             | 0 3 1 4 |                  |  |   |  |                  |  |               |  |
| 37                             | 0 4 7 1 |                  |  |   |  |                  |  |               |  |
| 49                             | 0 6 2 8 |                  |  |   |  |                  |  |               |  |
| 61                             | 0 7 8 5 | 73 80            |  |   |  |                  |  |               |  |
| 1                              | 3 6     | [Shaded Box]     |  |   |  |                  |  |               |  |
| 13                             | 0 9 4 2 |                  |  |   |  |                  |  |               |  |
| 25                             |         |                  |  |   |  |                  |  |               |  |
| 37                             |         |                  |  |   |  |                  |  |               |  |
| 49                             |         |                  |  |   |  |                  |  |               |  |
| 61                             |         | 73 80            |  |   |  |                  |  |               |  |
| 1                              | 4 1     | [Shaded Box]     |  | Minus sign indicates last card in first set of data |  |                  |  |               |  |
| 13                             | 3       |                  |  |   |  |                  |  |               |  |
| 25                             |         |                  |  |   |  |                  |  |               |  |
| 37                             |         |                  |  |   |  |                  |  |               |  |
| 49                             |         |                  |  |   |  |                  |  |               |  |
| 61                             |         | 73 80            |  |   |  |                  |  |               |  |
| FORM 114-C-17 REV. 7-55-VELLUM |         | 8                |  |   |  |                  |  |               |  |

Figure 5. Sample Data Sheets (Sheet 2 of 5)

DECK NO. \_\_\_\_\_ PROGRAMMER \_\_\_\_\_ DATE \_\_\_\_\_ PAGE 3 of 5 JOB NO. \_\_\_\_\_

| NUMBER | IDENTIFICATION | DESCRIPTION DO NOT KEY PUNCH   |
|--------|----------------|--|
| 1      | 73 80          |  |
| 13     |                |  |
| 25     |                | 4.2  |
| 37     |                | 2  |
| 49     |                |  |
| 61     |                |  |
| 1      | 9              |  |
| 13     | 73 80          |  |
| 25     |                | 5.1  |
| 37     |                | 1.0  |
| 49     |                |  |
| 61     |                |  |
| 1      |                | 1.0  |
| 13     | 73 80          | Minus sign indicates last card for first mode.   |
| 25     |                |  |
| 37     |                | Coefficients of the deflection polynomial on the tip. A vertical deflection of 1.0 causes a normal deflection of $\cos 30^\circ = 0.866$ . |
| 49     |                |  |
| 61     |                |  |
| 1      |                | 1.1  |
| 13     | 73 80          |  |
| 25     |                | 5.1  |
| 37     |                | 0.0  |
| 49     |                | 0.2  |
| 61     |                |  |
| 1      |                | 1.2  |
| 13     | 73 80          |  |
| 25     |                |  |
| 37     |                |  |
| 49     |                |  |
| 61     |                |  |
| 1      |                |  |

**Figure 5. Sample Data Sheets (Sheet 3 of 5)**

# FORTRAN FIXED IO DIGIT DECIMAL DATA

DECK NO. \_\_\_\_\_ PROGRAMMER \_\_\_\_\_ DATE \_\_\_\_\_ PAGE 4 of 5 JOB NO. \_\_\_\_\_

| NUMBER | IDENTIFICATION | DESCRIPTION DO NOT KEY PUNCH       |
|--------|----------------|------------------------------------|
| 1      | 73. 80         |                                    |
| 13     |                |                                    |
| 25     |                |                                    |
| 37     |                |                                    |
| 49     |                |                                    |
| 61     |                |                                    |
| 1      | 73. 80         |                                    |
| 13     |                |                                    |
| 25     |                |                                    |
| 37     |                |                                    |
| 49     |                |                                    |
| 61     |                |                                    |
| 1      | 73. 80         | JD                                 |
| 13     |                |                                    |
| 25     |                | NPTSW                              |
| 37     |                |                                    |
| 49     |                | NPTST                              |
| 61     |                |                                    |
| 1      | 73. 80         |                                    |
| 13     |                |                                    |
| 25     |                |                                    |
| 37     |                |                                    |
| 49     |                |                                    |
| 61     |                |                                    |
| 1      | 73. 80         | Deflection data on the wing begin. |
| 13     |                | X1                                 |
| 25     |                | S1                                 |
| 37     |                | N1                                 |
| 49     |                | X2                                 |
| 61     |                | S2                                 |
| 1      | 73. 80         |                                    |
| 13     |                |                                    |
| 25     |                |                                    |
| 37     |                |                                    |
| 49     |                |                                    |
| 61     |                |                                    |
| 1      | 73. 80         |                                    |
| 13     |                |                                    |
| 25     |                |                                    |
| 37     |                |                                    |
| 49     |                |                                    |
| 61     |                |                                    |

Figure 5. Sample Data Sheets (Sheet 4 of 5)

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# FORTRAN FIXED 10 DIGIT DECIMAL DATA

DECK NO. \_\_\_\_\_ PROGRAMMER \_\_\_\_\_ DATE \_\_\_\_\_ PAGE 5 of 5 JOB NO. \_\_\_\_\_

| NUMBER | IDENTIFICATION | DESCRIPTION DO NOT KEY PUNCH                         |
|--------|----------------|--|
| 1      | 1.6 6          | End of deflection data on the wing                   |
| 13     | 1 2.0 6        | N22  |
| 25     |                |  |
| 37     |                |  |
| 49     |                |  |
| 61     | 73. 80         |  |
|        | 2.6            |  |
| 1      | 3.0 1          | Deflection data on the tip begin                     |
| 13     | 4. 2 5         | X1   |
| 25     | 2. 3           | S1   |
| 37     | 1. 3 8         | N1   |
| 49     | 4. 5           | X2   |
| 61     | 2. 3           | S2   |
|        | 2.7            |  |
| 1      | 3.1 6          | End of deflection data on the tip                    |
| 13     | 4. 8           | X6   |
| 25     | 2. 7           | S6   |
| 37     | 1. 4 4         | N6   |
| 49     |                |  |
| 61     | 73. 80         |  |
|        | 3.0            |  |
| 1      | 9.8            |  |
| 13     | 1              | Indicator that modes are not to be read in hereafter |
| 25     |                |  |
| 37     |                |  |
| 49     |                |  |
| 61     | 73. 80         |  |
|        | 3.1            |  |

FORM 114-C-17 REV. 7-53-YELLUM

Figure 5. Sample Data Sheets (Sheet 5 of 5)

## VII. RESULTS

The computer program was applied to the rigid modes of two wings.

### ASPECT RATIO 2.0 RECTANGULAR WING FOLDED AT 80 PERCENT SEMISPAN

$C_{L\alpha}$  is plotted as a function of Mach number in Figure 6. The dashed curve is a plot of the approximation

$$C_{L\alpha} = \frac{2\pi A.R.}{2 + \sqrt{4 + (A.R.)^2 (1-M^2)}}$$

where A.R. is the aspect ratio. This is formula (6-31) of Reference 5, for the case of a rectangular wing.

### TRIANGULAR WING WITH FOLDED TIPS

The configuration used was a triangular wing with a sweep angle of 65 degrees, folded at 60 percent semispan.

Figures 7 and 8 show  $C_{L\alpha}$  and  $C_{m\alpha}$  as functions of fold angle (at  $M = 0.8$ ), and as functions of Mach number. Figure 9 is a plot of unsteady generalized forces for rigid oscillations of the wing in the pitching mode.

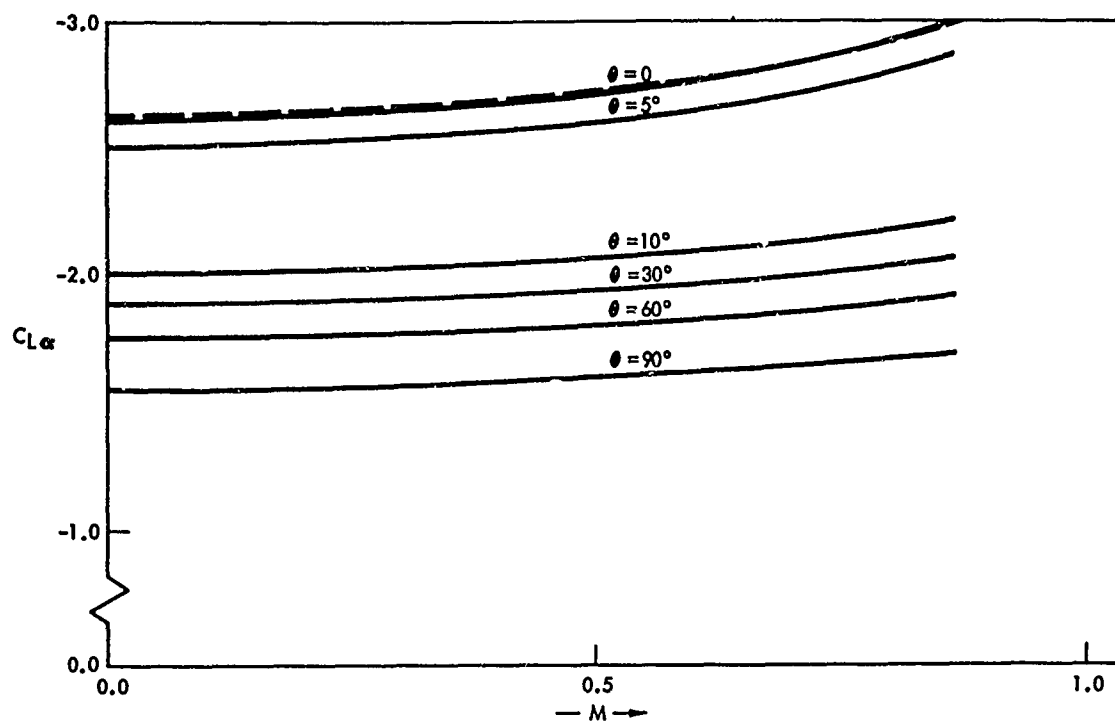


Figure 6. Lift and Moment Coefficients Vs Mach Number for Aspect Ratio 2.0 Rectangular Wing at Various Fold Angles

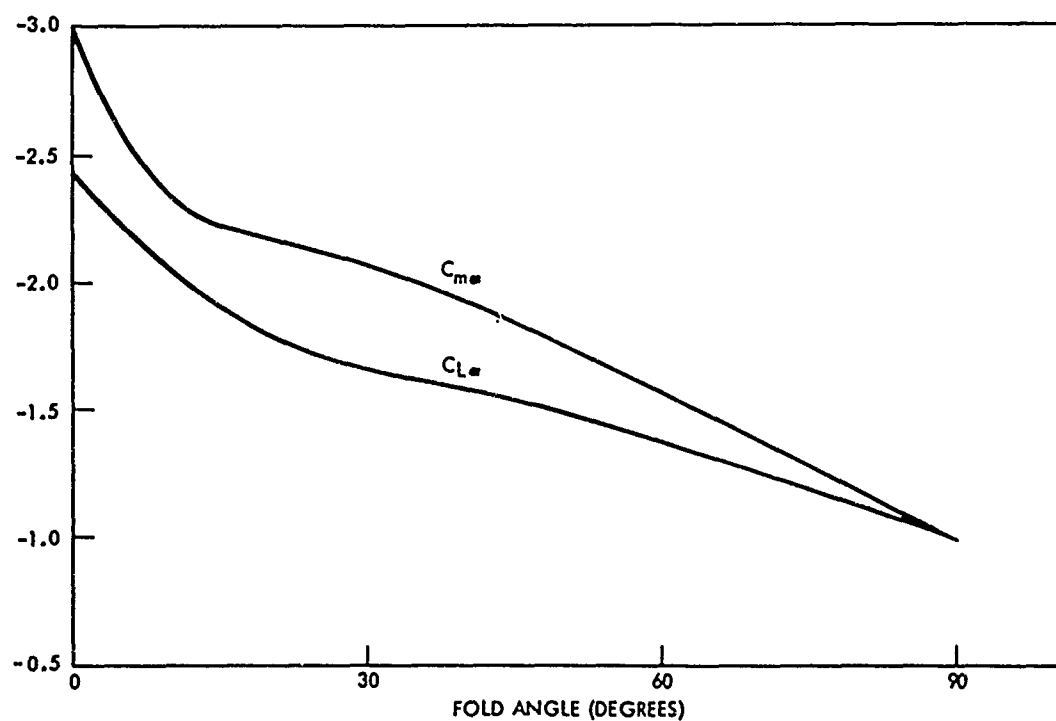


Figure 7. Lift and Moment Coefficients Vs Fold Angle for  $65^\circ$  Triangular Wing at  $M_\infty = 0.8$

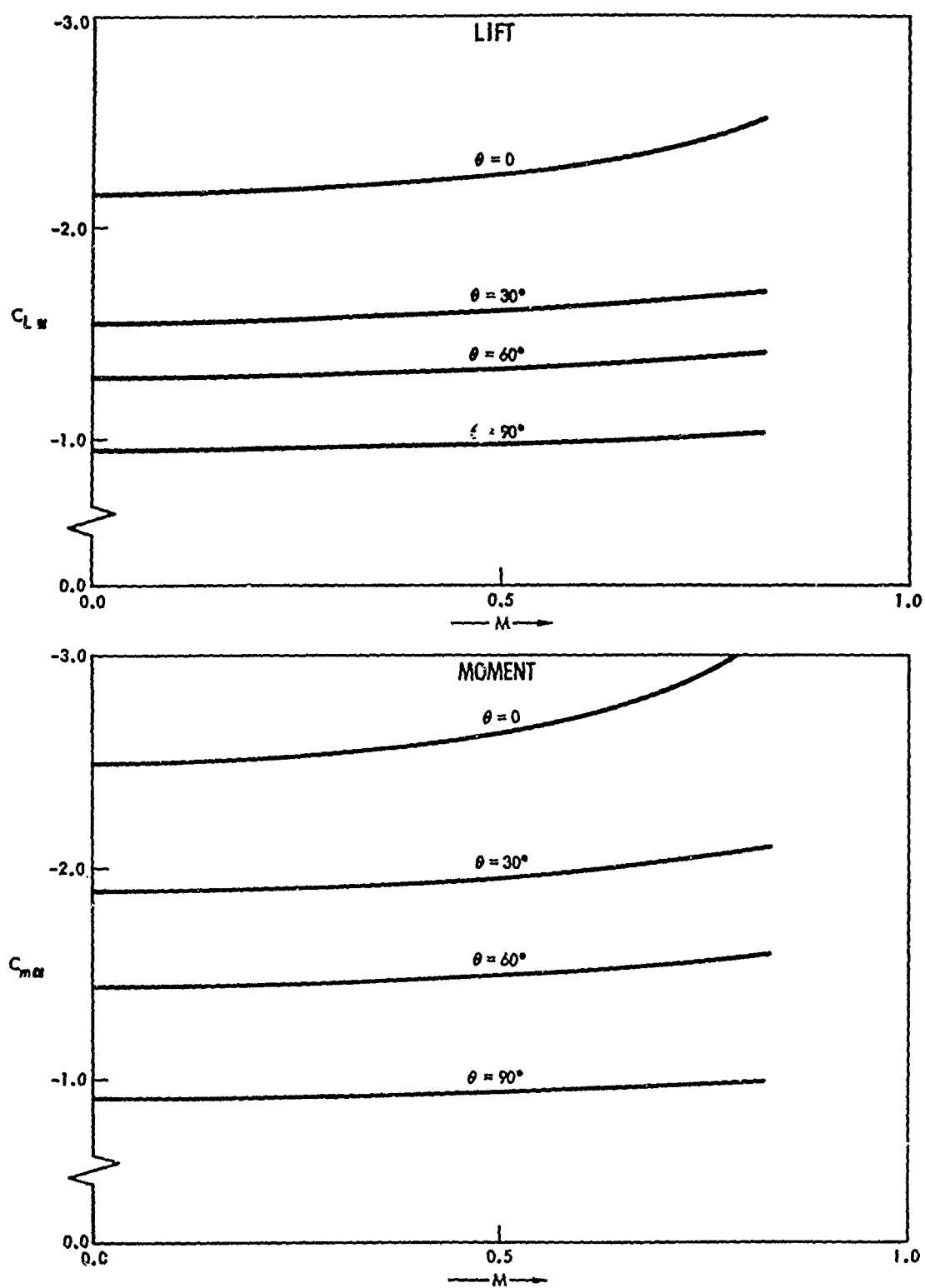
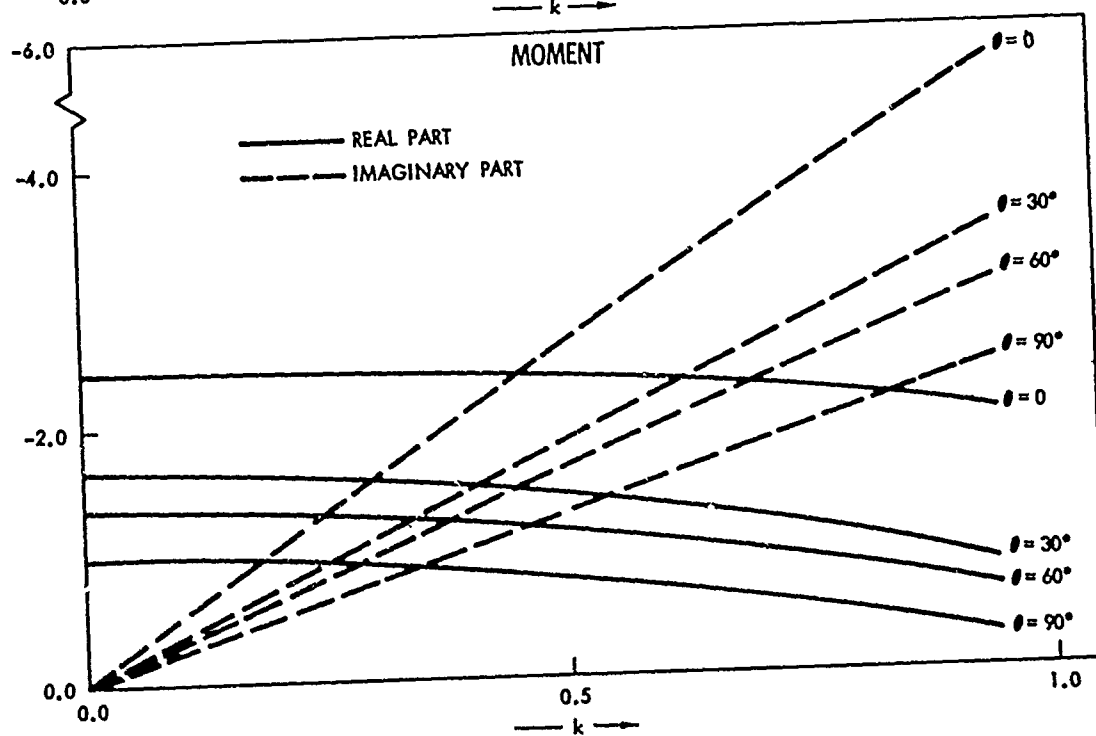
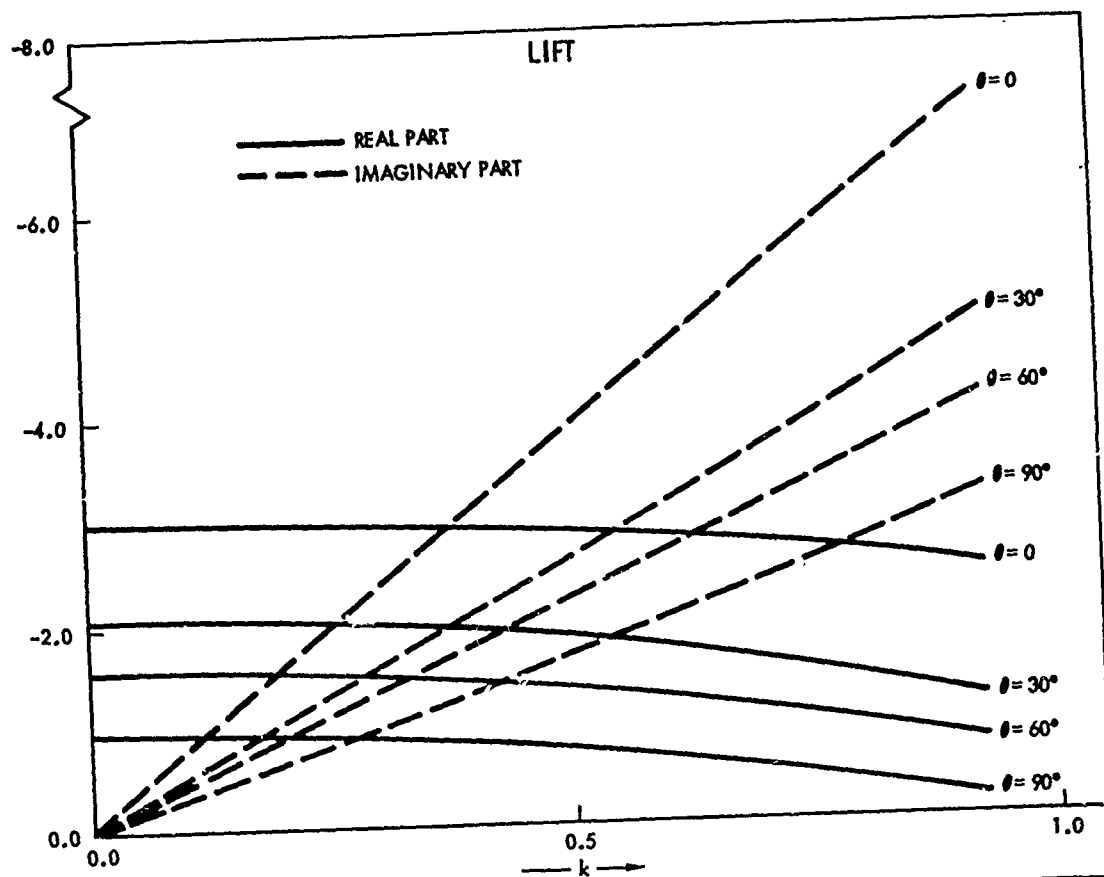


Figure 8. Lift and Moment Coefficients Vs Mach Number for 65° Triangular Wing at Various Fold Angles



Coefficients of Lift and Moment Due to Pitch Vs Reduced Frequency for a  
65° Triangular Wing at  $M_\infty = 0.8$  and Various Fold Angles

## VIII. CONCLUSIONS AND RECOMMENDATIONS

The results obtained by the nonplanar kernel function method show the expected trend with increasing fold angle, which agrees with the observed experimental trend. For zero fold angle, the method reduces to that already used by Hsu (Reference 4).

Possible extensions of the method include the treatment of more general configurations, or of other specific configurations, such as the T-tail. Also, any generalizations proposed for the planar case should be considered here, such as the problem of a nonplanar wing with a control surface.

The formula that was used for the kernel function (page 19) should be useful in any future developments using the three-dimensional kernel function. The previously available formula was much longer. A special case of that formula is given in Reference 12. The use of the simplified kernel function, together with Hsu's method of integration, results in greatly reduced computer running times. This makes it practical to use the kernel function method as a tool in the preliminary analysis of new wing configurations.

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APPENDIX. COMPUTER PROGRAM LISTINGS

MP

```

COMMON W(50,25),WI(50,25),A(2,25,25),B(2,25,10),XSV(10,5)
COMMON X(10),Y(10),PSI(10),ETA(10),ZXY(21,10),ZZZ(21,10)
COMMON /C0M1/ N,M,NR,NC,NDATA,NM0D,NSYM,NDEX,NCC,NCS1,NCST
COMMON/C0M1/FM,FR,BO,SF,ST
COMMON /CDAT/DAT(500)
DO 1 I=1,100
1 DAT(I)=0.0
2 CALL DATRD(DAT)
CALL DATA
DO 200 ND=1,NDATA
FR=DAT(ND+30)
FM=DAT(ND+20)
CALL DATA2
CALL ZEN
DO 35 I=1,NC
DO 35 J=1,NC
A(1,I,J)=0.0
A(2,I,J)=0.0
DO 35 K=1,NR
A(1,I,J)=A(1,I,J)+W(K,I)*W(K,J)+WI(K,I)*WI(K,J)
A(2,I,J)=A(2,I,J)+W(K,I)*W(K,J)-WI(K,I)*W(K,J)
35 CONTINUE
K=MSIMEC(25,NC,NM0D,A,B)
IF (K.NE.1) GO TO 99
WRITE (6,650)
650 FORMAT(1H18X,78HPRESSURE = STIP/B(S) * SUM OF A(N,M)*F(N,PSI)*SQRT00000270
1(1.0-(S/STIP)**2)*(S/STIP)**)
IF (NSYM.LT.0) GO TO 10
WRITE (6,651)
651 FORMAT(1H+86X,5H(M-1))
GO TO 15
10 WRITE (6,652)
652 FORMAT(1H+86X,1HM)
15 LINES=1
LINLIM=35-NC/2
DO 40 I1=1,NM0D
WRITE (6,700) I1

```

00000020  
00000030  
00000040  
00000050  
00000060  
00000070  
00000080  
00000090  
00000100  
00000110  
00000120  
00000130  
00000140  
00000150  
00000160  
00000170  
00000180  
00000190  
00000200  
00000210  
00000220  
00000230  
00000240  
00000250  
00000260  
00000270  
00000280  
00000290  
00000300  
00000310  
00000320  
00000330  
00000340  
00000350  
00000360  
00000370  
00000380

MP

```

700 FORMAT(1H-20X,41HPRESSURE COEFFICIENTS A(N,M) FOR MODE NO.12/1H0
1 2(9X,5HN M6X,9HREAL PART7X,10HIMAG PART )/1H )
J1=1
JCARR=1
DO 19 NPR=1,N
DO 19 MPR=1,M
IF (JCARR.EQ.0) GO TO 17
JCARR=0
WRITE (6,705) NPR,MPR,B(1,J1,I1),B(2,J1,I1)
GO TO 19
17 JCARR=1
WRITE (6,706) NPR,MPR,B(1,J1,I1),B(2,J1,I1)
705 FORMAT(111,14,1P2E16.5)
706 FORMAT(1H+52X,2I4,1P2E16.5)
19 J1=J1+1
LINES=LINES+6*NC/2
IF (LINES.LE.LINLIM) GO TO 40
WRITE (6,708)
708 FORMAT(1H1)
LINES=0
40 CONTINUE
IF (LINES.EQ.0) GO TO 24
LINES=LINES+7+2*NM0D**2
IF (LINES.LE.40) GO TO 23
WRITE (6,708)
GO TO 24
23 WRITE (6,709)
709 FORMAT(1H0)
24 WRITE (6,45) FR,FM,DAT(10)
45 FORMAT(1H010X,26HGENERALIZED FORCES - - K =F7.4,6H, M =F7.4,
1 15H, FOLD ANGLE =F8.4,5H DEG.)
WRITE (6,46)
46 FORMAT(1H05X,5HMGDES/4X,11H0SC. DEFL.8X,9HREAL PART10X,9HIMAG PAR00000710
1T10X,10HABS. VALUE6X,11HPHASE ANGLE)
CALL FORCE
200 CONTINUE
GO TO 2

```

MP

99 WRITE (6,98), ND  
98 FORMAT(57H1 PRESSURE COEFFICIENTS CANNOT BE FOUND FOR DATA CASE  
1.12)  
GO TO 200  
END

00000760  
0000000770  
00000780  
00000790  
00000800

# DATA1

## SUBROUTINE DATA1

```

COMMON W(50,25),WI(50,25),A(2,25,25),B(2,25,10),XSV(10,5)
COMMON X(10),Y(10),PSI(10),ETA(10),ZXY(21,10),ZZZ(21,10)
COMMON /C0M1/ N,M,NR,NC,NDATA,NM0D,NSYM,NDEX,NCC,NCS1,NCST
COMMON /C0M1/ FM,FR,BO,SF0LD,STIP
COMMON /C0M2/ Q(5),DELP(5,10),NIC,NIS,ZIS,ZIP,ZCP,SF0ST
COMMON /C0M2/ BF,BMF,C0S1,SIN1,CT32P,CT32M,CT54P,CT54M
COMMON /CDAT/DAT(500)
COMMON /C0NST/ PI,PI0VR2,RAD,ZI(10)

```

N=DAT(1)

M=DAT(2)

NCC=DAT(3)

NCS=DAT(4)

NDATA=DAT(5)

NSYM=DAT(6)

```

NCS = N0. 0F COLLOCATION PTS. SPANWISE
NCC = N0. 0F COLLOCATION PTS. CHORDWISE
NIS = N0. 0F INTEGRATION PTS. SPANWISE
NIC = N0. 0F INTEGRATION PTS. CHORDWISE
NDEX = (1-NSYM)/2

```

NIC = NCC

NIS = NCS + 1

ZIS = NIS

BO=DAT(9)

SF0LD=DAT(7)/BO

STIP=DAT(8)/BO

00 15 I=1,10

X(I)=D0TS

Y(I)=D0TS

PSI(I)=D0TS

ETA(I)=D0TS

SB02=STIP\*\*2

WRITE (6,20) NCS,NCC,M,N,NDATA,NDATA

```

20  FORMAT(1H1,54X,10HINPUT DATA/21X,16HCOLLOCATION PTS., 5X,14HPRESSU00001150
1RE MODES, 5X, 9HMACH NOS., 5X, 6HFREQS./21X,12,5H SPAN,13,6H CHORD00001160
2,4X,12,5H SPAN,13,6H CHORD,9X,12,11X,12)
13  SF0ST = SF0LD/STIP
00001170
00001180

```

DATA1

ALFA1=DAT(10)/RAD  
 ALFA2=(90.0-DAT(11))/RAD  
 ALFA3=(90.0-DAT(12))/RAD  
 ALFA4=(90.0-DAT(13))/RAD  
 ALFA5=(90.0-DAT(14))/RAD  
 EVALUATE TIP INTEGRALS  
 F= SQRT(1.0-SF0ST\*\*2)  
 THE1=ATAN(SF0ST/F)  
 Q(1)=1.0-(THE1+F\*SF0ST)/PI0VR2  
 F=F\*\*3/PI  
 Q(2)=F/3.0  
 DO 1607 IL=3,5  
 F=F\*SF0ST  
 Q(IL)=(F+ZI(IL-2)\*Q(IL-2))/ZI(IL+1)

C

1607 CONTINUE  
 C GENERATE COLLOCATION AND INTEGRATION STATIONS

ZCP=2.0\*PI/FL0AT(2\*NCC+1)

ZIR=S802\*ZCP/8.0

F=0.5\*ZCP

DO 27 J=1,NCC

PSI(J)=-COS(F)

W=PSI(J)

DO 26 I=1,N

26 DELP(I,J)=FXI(W,I)\*ZIR

I=NCC+1-J

X(I)=-W

27 F=F+ZCP

ZS=PI/ZIS

F=-ZS+PI0VR2

DF=0.5\*ZS

DO 29 J=1,NCS

Y(J)=SIN(F)

W=F+DF

ETA(J)=SIN(W)

29 F=F-ZS

ETA(NIS)=-ETA(1)

WRITE ( 6,35)

00001190  
 00001200  
 00001210  
 00001220  
 00001230  
 00001235  
 00001240  
 00001250  
 00001260  
 00001270  
 00001280  
 00001290  
 00001300  
 00001310  
 00001320  
 00001330  
 00001340  
 00001350  
 00001360  
 00001370  
 00001380  
 00001390  
 00001400  
 00001410  
 00001420  
 00001430  
 00001440  
 00001450  
 00001460  
 00001470  
 00001480  
 00001490  
 00001500  
 00001510  
 00001520  
 00001530  
 00001540

# DATA1

```

35  FORMAT(1H0,27X,47H,CALCULATED COLLOCATION AND INTEGRATION STATIONS/00001550
    1H ,24X,1HX,15X,3HPSI,15X,1HY,15X,3HETA)
    NLIN=MAX0(NIS,NCC)
    WRITE (6,36) (X(I),PSI(I),Y(I),ETA(I),I=1,NLIN)
36  FORMAT(13X,4F17.4)
    WRITE (6,560)
    WRITE (6,565) (DAT(I+20),DAT(I+30),I=1,NDATA)
    C
    EVALUATE QUANTITIES FOR LATER USE
    C0S1=C0S(ALFA1)
    SIN1=SIN(ALFA1)
    NCS1=(NCS+1)/2
    IF (NSYM.LT.0) NCS1=NCS/2
    CTN2=0.5*C0S(ALFA2)/SIN(ALFA2)
    CTN3=0.5*C0S(ALFA3)/SIN(ALFA3)
    CTN4=0.5*C0S(ALFA4)/SIN(ALFA4)
    CTN5=0.5*C0S(ALFA5)/SIN(ALFA5)
    CT32P=CTN3+CTN2
    CT32M=CTN3-CTN2
    CT54P=CTN5+CTN4
    CT54M=CTN5-CTN4
    BF =1.0+SF0LD*CT32M
    BMF=1.0+SF0LD*CT32P
    NR=NCC*NCS1
    NC=M*N
    DO 82 J=1,10
    IF (Y(J).LE.SF0ST) GO TO 84
82  CONTINUE
84  NCST=J-1
    ZIP=16.0/SB02
    RETURN
560  FORMAT(1H0,35X, 9HMACH NOS. ,15X,6HFREQS.)
565  FORMAT(1H ,36X,F8.4,14X,F8.4)
    END

```

DAT2

SUBROUTINE DATA2

```

COMMON WASH(50,25),WASHI(50,25),A(2,25,25),B(2,25,10),XSV(10,5)
COMMON X(10),Y(10),PSI(10),ETA(10),ZXY(21,10),ZZ(21,10)
COMMON /COM1/ N,M,NR,NC,NDATA,NMOD,NSYM,NDEX,NCC,NCS1,NCST
COMMON /COM1/ FM,FR,BO,SFOLD,STIP
COMMON /COM2/ Q(5),DELP(5,10),NIC,NIS,ZIS,ZIP,ZCP,SFQST
COMMON /COM2/ BF,BMF, C0S1,SIN1,CT32P,CT32M,CT54P,CT54M
COMMON /CDAT/DAT(500)
COMMON /CONST/ PI,PIQVR2,RAD,ZI(10)
DIMENSION RA(10),RB(10),RC(5),FS(5),FT(5),FU(5),FV(5),FW(5),FX(5)
DIMENSION XU(2),XX(2),XY(2),XZ(2)
EVALUATION OF MATRIX OF INTEGRALS
DO 38 J=1,1250
  WASH(J,1) = 0.0
  WASHI(J,1) = 0.0
  BETA2=1.0-FM**2
  IU = 0
  DO 1000 IQ=1,NCS1
    CALL GEOM(Y(IQ),XI1,XI2,C02,SI2,YY,ZY)
    DO 1000 IP = 1,NCC
      XA=X(IP)*XI2+XI1
      XSV(IP,IQ) = XA
      IU = IU+1
      XARG=ZI(IP)*ZCP
      F=SQRT(1.0-X(IP)**2)
      RC(1)=2.0*(F+XARG)
      G=-2.0*X(IP)*F
      RC(2)=XARG-0.5*G
      DO 1604 IC=3,N
        H=-F-2.0*X(IP)*G
        RC(IC)=F/ZI(IC-2)-H/ZI(IC)
        F=G
        G=H
      1604 G=H
      58 DO 1640 IN = 1,NIS
        YE=ETA(IN)
        CALL GEOM(YE,XI3,XI4,C04,SI4,YETA,ZETA)
        SY=1.0

```

00001890  
00001900  
00001910  
00001920  
00001930  
00001940  
00001950  
00001960  
00001970  
00001980  
00001990  
00001995  
00002000  
00002010  
00002020  
00002030  
00002040  
00002050  
00002060  
00002070  
00002080  
00002090  
00002100  
00002110  
00002120  
00002130  
00002140  
00002150  
00002160  
00002170  
00002180  
00002190  
00002200  
00002210  
00002220  
00002230  
00002240

C

38

1604  
58

DAT2

IF (YE.GE.0.0) GO TO 54  
SY=SIGN(1.0,NSYM)  
YE=SIGN(YE,NSYM)

54 CONTINUE

YO=YY-YETA  
ZOO=ZY-ZETA  
DO 1580 IJ=1,N  
FS(IJ) = 0.0  
FT(IJ) = 0.0  
FU(IJ) = 0.0

1580 FV(IJ)=0.0

DO 1600 IL = 1,NIC  
XB=PSI(IL)\*XI4+XI3

XO = XA - XB

51 CALL KRNL1(FR,XO,YO,ZOO,FM,BETA2,C02,SI2,C04,SI4,XX)  
IF (SI4.EQ.0.0) GO TO 93

C ETA IS OUTBOARD OF FOLDLINE,CALCULATE KPRIME.

96 IF(ETA(IN))I01,I01,I02

101 SA=YY+SFOLD

GO TO 103

102 SA=YY-SFOLD

103 X1=XA-PSI(IL)\*BF-BMF

CALL KRNL1(FR,X1,SA,ZY,FM,BETA2,C02,SI2,C04,SI4,XY)

CALL KRNL1(FR,X1,SA,ZY,FM,BETA2,C02,SI2,1.0,0.0,XZ)

C KPRIME = XX - XY + XZ

XU(1) = XY(1) - XZ(1)

XU(2) = XY(2) - XZ(2)

XX(1) = XX(1) - XU(1)

XX(2) = XX(2) - XU(2)

GO TO 94

93 XU(1) = 0.0

XU(2) = 0.0

94 DO 1590 IJ=1,N

FS(IJ)=FS(IJ)+XX(1)\*DELP(IJ,IL)

FT(IJ)=FT(IJ)+XX(2)\*DELP(IJ,IL)

FU(IJ)=FU(IJ)+XU(1)\*DELP(IJ,IL)

1590 FV(IJ)=FV(IJ)+XU(2)\*DELP(IJ,IL)

00002250  
00002260  
00002270  
00002280  
00002290  
00002300  
00002310  
00002320  
00002330  
00002340  
00002350  
00002360  
00002370  
00002380  
00002390  
00002400  
00002410  
00002420  
00002430  
00002440  
00002450  
00002460  
00002470  
00002480  
00002490  
00002500  
00002510  
00002520  
00002530  
00002540  
00002550  
00002560  
00002570  
00002580  
00002590  
00002600  
00002610

DAT2

1600 CONTINUE  
C CHORDWISE INTEGRATION IS COMPLETE.

IA = 0  
W=(1.0-YE\*\*2)/ZIS  
IF (NDEX.NE.0) W=W\*YE  
DO 1625 IL=1,M  
ILN=IL+NDEX  
DO 1620 IM = 1,N  
IA = 1 + IA  
WASH(IU,IA)=WASH(IU,IA)+FS(IM)\*W  
WASHI(IU,IA)=WASHI(IU,IA)+FT(IM)\*W  
IF (IN.NE.1.AND .IN.NE.NIS) GO TO 1620  
WASH(IU,IA)=WASH(IU,IA)+FU(IM)\*Q(ILN)\*SY  
WASHI(IU,IA)=WASHI(IU,IA)+FV(IM)\*Q(ILN)\*SY

1620 CONTINUE

1625 W=YE\*W

1640 CONTINUE

DO 1601 LG=1,IP  
ARG=FR\*(XA-PSI(LG)\*XI2-XI1)  
RA(LG)=ZIP\*(COS(ARG)-1.0)  
RB(LG)=-ZIP\*SIN(ARG)

1601 CONTINUE

C RA = REAL, RB = IMAG

DO 1646 IJ=1,N

FW(IJ) = 0.0

FX(IJ) = 0.0

DO 1646 IK=1,IP

FW(IJ) = FW(IJ) + DELP(IJ,IK)\*RA(IK)

FX(IJ) = FX(IJ) + DELP(IJ,IK)\*RB(IK)

1646 CONTINUE

IA = 0

YP=ZIS/8.0

IF (NDEX.NE.0) YP=YP\*Y(IQ)

DO 1649 IL = 1,M

DO 1648 IM = 1,N

IA = 1+IA

WASH(IU,IA) = WASH(IU,IA) - (FW(IM)+RC(IM))\*YP

00002620  
00002630  
00002640  
00002650  
00002660  
00002670  
00002680  
00002690  
00002700  
00002710  
00002720  
00002730  
00002740  
00002750  
00002760  
00002770  
00002780  
00002790  
00002800  
00002810  
00002820  
00002830  
00002840  
00002850  
00002860  
00002870  
00002880  
00002890  
00002900  
00002910  
00002920  
00002930  
00002940  
00002950  
00002960  
00002970  
00002980

DATA2

```

1648 WASHI(IU,IA) = WASHI(IU,IA)-FX(IM)*YP
1649 YP=YP*Y(IQ)
1000 CONTINUE
1010 WRITE (6,530) FR,FM
      DO 1015 K = 1,IU
      WRITE (6,555)
      WRITE (6,535)(WASH(K,L),WASHI(K,L),L=1,IA)
1015 CONTINUE
      RETURN
530 FORMAT(1H12IX,30HMATRIX FOR REDUCED FREQUENCY =F7.4,
1 16H, MACH NUMBER =F7.4)
535 FORMAT(3(5X,1P2E15.4,1HI))
555 FORMAT(1H )
      END

```

00002990  
00003000  
00003010  
00003020  
00003030  
00003040  
00003050  
00003060  
00003080  
00003090  
00003100  
00003110  
00003120  
00003130

GEOM

```

SUBROUTINE GEOM(Y,X1,X2,C0,SI,YY,ZY)
COMMON /C0M1/ N,M,NR,NC,NDATA,NM0D,NSYM,NDEX,NCC,NCS1,NCST
COMMON /C0M1/FM,FR,BO,SF0LD,STIP
COMMON /C0M2/ Q(5),DELP(5,10),NIC,NIS,Z'IS,ZIP,ZCP,SF0ST
COMMON /C0M2/BF,BMF, C0S1,SIN1,CT32P,CT32M,CT54P,CT54M
EVALUATION OF GEOMETRICAL QUANTITIES AT A GIVEN CHORD
Y1=STIP*ABS(Y)
F=Y1-SF0LD
IF (F) 2,2,4
2 C0=1.0
SI=0.0
YY=Y1
ZY=0.0
X1=1.0+Y1*CT32P
X2=1.0+Y1*CT32M
GO TO 6
4 C0=C0S1
SI=SIN1
YY=SF0LD+F*C0S1
ZY=F*SIN1
X1=BMF+F*CT54P
X2=BF+F*CT54M
6 IF (Y.GE.0.0) RETURN
SI=-SI
YY=-YY
RETURN
END

```

00003150  
00003160  
00003170  
00003180  
00003190  
00003195  
00003200  
00003210  
00003220  
00003230  
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00003250  
00003260  
00003270  
00003280  
00003290  
00003300  
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00003320  
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00003350  
00003360  
00003370  
00003380  
00003390  
00003400

KRNL

```

SUBROUTINE KRNL(C,X,Y,ZO,CM,B2,C01,S11,C02,S12,AKERN)
DIMENSION Z(6),H(6),AKERN(2)
DATA H/0.08566225,0.18038079,2*0.23395697,0.18038079,0.08566225/
DATA Z/0.96623476,0.83060469,0.61930959,0.38069041,0.16939531,
10.03376524/
R2 = Y*Y+Z0*Z0
R = SQRT(R2)
CK1 = CK*R
G1 = 0.0
G2 = 0.0
G3 = 0.0
G4 = 0.0
G5 = 0.0
G6 = 0.0
S2 = X*X + B2*R2
S = SQRT(S2)
U1 = (CM*S-X)/(B2*R)
UK = CK1*U1
DO 20 I = 1,6
UZ = U1*Z(I)
UZ2 = UZ**2
F = UK*Z(I)
C0 = COS(F)
S1 = SIN(F)
F = H(I)/ SQRT(1.0+UZ2)*UZ*U1
G3 = G3 + F*C0
G4 = G4 - F*S1
F = UZ*F
G5 = G5 + F*C0
G6 = G6 - F*S1
V = 1.0 - Z(I)**2
F = H(I)*2.0*V* EXP(-CK1*V)/ SQRT(1.0+V)
G1 = G1 + F
G2 = G2 + V*F
G7 = G1 + G3
XS = X/S
C0 = COS(UK)*XS

```

5

20

KRNL

```

SI = SIN(UK)*XS
IF(CK.NE.0.0) GO TO 22
F14 = 1.0
F15 = 2.0
GO TO 23
22 F14 = CK1*BESEL(CK1,1,3)
    F15 = CK1*CK1*BESEL(CK1,2,3)
23 F11 = -(CK1*G4+F14+C0)
    F12 = CK1*G7+SI
    F = 2.0+B2*R2/S2
    FP = UK+CK*CM*R2/S
    F21 = CK1*G4+CK1*CK1*G5+F15+C0*F +SI*FP
    F22 = CK1*(-G7+CK1*(G6-G2))+C0*FP-SI*F
    XK = CK*X
    C0 = C0S(XK)
    SI = SIN(XK)
    F=(C01*C02+SI1*SI2)/R2
    FP=(Z0*C01-Y*SI1)*(Z0*C02-Y*SI2)/R2**2
    G1 = F*F11+FP*F21
    G2 = F*F12+FP*F22
    AKERN(1) = -C0*G1-SI*G2
    AKERN(2) = SI*G1-C0*G2
    RETURN
    END
00003790
00003800
00003810
00003820
00003830
00003840
00003850
00003860
00003870
00003880
00003890
00003900
00003910
00003920
00003930
00003940
00003950
00003960
00003970
00003980
00003990
00004000
00004010
00004020

```

ZEN

```

SUBROUTINE ZEN
COMMON W(50,25),WI(50,25),A(2,25,25),B(2,25,10),XSV(10,5)
COMMON X(10),Y(10),PSI(10),ETA(10),ZXY(21,10),ZZZ(21,10)
COMMON /COM1/ N,M,NR,NC,NDATA,NMOD,NSYM,NDEX,NCC,NCS1,NCST
COMMON /COM1/ FM,FR,BO,SF,ST
COMMON /CDAT/DAT(500)
DIMENSION XH(66),SH(66),ZH(66),XCH(50),YCH(50),Z(50),DZ(50)
DIMENSION WD(2,100)
EQUIVALENCE (XH,A),(SH,A(67)),(ZH,A(133)),(XCH,A(200)),(Z,A(250))
EQUIVALENCE (YCH,A(300)),(DZ,A(350)),(WD,A(400))
LINES=50
NMOD=DAT(41)
IDEF=0
DO 50 I=1,NMOD
IF (DAT(98).NE.0.0) GO TO 20
IF (IDEF.NE.0) GO TO 20
CALL DATRD(DAT)
JD=DAT(42)
IF (JD-1) 2,4,16
2 IDEF=1
GO TO 20
4 READ DEFLECTION VALUES AT POINTS
L=101
DO 8 J=1,NPTS
XH(J)=DAT(L)
SH(J)=DAT(L+1)
ZH(J)=DAT(L+2)
8 L=L+3
NDH=5
9 CALL LSSURP(XH,SH,ZH,NPTS,NDH,G,ZXY(1,1),IND)
IF (IND.EQ.0) GO TO 10
NDH=NDH-1
GO TO 9
10 NPTS=DAT(44)
L=301
DO 11 J=1,NPTS

```

ZEN

XH(J)=DAT(L)  
SH(J)=DAT(L+1)  
ZH(J)=DAT(L+2)

11 L=L+3  
NDV=5

12 CALL LSSURP(XH,SH,ZH,NPTS,NDV,G,ZZZ(1,I),IND)  
IF (IND.EQ.0) GO TO 20  
NDV=NDV-1  
GO TO 12

C REA: POLYNOMIAL COEFFICIENTS

16 DO 17 J=1,21  
ZXY(J,I)=DAT(J+50)  
17 ZZZ(J,I)=DAT(J+75)

C FIND UPWASHES

20 K=0  
DO 24 I2=1,NCST  
DO 24 J=1,NCC  
K=K+1  
XCH(K)=80\*XSX(J,I2)  
24 YCH(K)=DAT(8)\*Y(I2)  
LIND=8+K

IF (LINES+LIND.LE.40) GO TO 25  
WRITE (6,41)  
41 FORMAT(1H1)

LINES=0  
25 LINES=LINES+LIND  
WRITE (6,43)

WRITE(6,42) I,ZZZ(J,I),J=1,21)  
42 FORMAT(1H+5X,26HDEFLECTION COEFFICIENTS ON7X,8HMODE NO.12/1H /  
1 (1P7E14.4))

43 FORMAT(1H032X,4HTIP,)  
CALL ZDZ(XCH,YCH,K,ZZZ(1,I),Z,DZ)  
DO 26 I2=1,K

WD(1,I2)=DZ(I2)\*80  
26 WD(2,I2)=FR\*Z(I2)  
I3=NCST+1  
L=0

00004410  
00004420  
00004430  
00004440  
00004450  
00004460  
00004470  
00004480  
00004490  
00004495  
00004500  
00004510  
00004520  
00004525  
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00004670  
00004680  
00004690  
00004700  
00004710  
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00004740  
00004750

ZEN

```

D0 27 I2=I3,NCS1
D0 27 J=1,NCC
L=L+1
XCH(L)=80*XS(V(J,I2))
27 YCH(L)=DAT(8)*Y(I2)
LIND=8+L
IF (LINES+LIND.LE.40) GO TO 28
WRITE (6,41)
LINES=0
28 LINES=LINES+LIND
WRITE (6,44)
44 FORMAT(1H032X,5HWING,)
WRITE (6,42) I,(ZXY(J,I),J=1,21)
CALL ZDZ(XCH,YCH,L,ZXY(1,I),Z,DZ)
J2=K+1
D0 29 I2=1,L
WD(1,J2)=DZ(I2)*80
WD(2,J2)=FR*Z(I2)
29 J2=J2+1
D0 33 J=1,NC
B(1,J,I)=0.0
B(2,J,I)=0.0
D0 33 K=1,NR
B(1,J,I)=B(1,J,I)+W(K,J)*WD(1,K)+WI(K,J)*WD(2,K)
33 B(2,J,I)=B(2,J,I)+W(K,J)*WD(2,K)-WI(K,J)*WD(1,K)
50 CONTINUE
RETURN
END

```

00004760  
00004770  
00004780  
00004790  
00004800  
00004810  
00004820  
00004830  
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00004920  
00004930  
00004940  
00004950  
00004960  
00004970  
00004980  
00004990  
00005000  
00005010  
00005020  
00005030

F0RC

SUBROUTINE F0RC

```

COMMON W(50,25),WI(50,25),A(2,25,25),AP(2,25,10),XSV(10,5)
COMMON X(10),Y(10),PSI(10),ETA(10),ZXY(21,10),ZZZ(21,10)
COMMON /CGM1/ N,M,NR,NC,NDATA,NMOD,NSYM,NDEX,NCC,NCS1,NCST
COMMON /CGM1/FM,FR,80,SF,ST
COMMON /CGM2/ Q(5),DELP(5,10),NIC,NIS,ZIS,ZIP,ZCP,SF0ST
COMMON /CGM2/BF,BMF, CGS1,SIN1,CT32P,CT32M,CT54P,CT54M
COMMON /CGNST/ PI,PI0VR2,RAD,ZI(10)
DIMENSION AW(5,6,10),AT(5,6,10),AS(10)
EQUIVALENCE (A,AW),(A(301),AT)
DIMENSION FRC(2)
DIMENSION U(6),V(6),H(6)
DATA H/0.085662246,0.18038079,2*0.23395697,0.18038079,0.085662246/
DATA V/0.96623476,0.83060459,0.61930959,0.38069041,0.16939531,
10.03376524/
C WEIGHTS AND POINTS FOR 6 POINT GAUSSIAN QUADRATURE ON (0,1)
DATA U/-0.965926,-0.707107,-0.258819,0.258819,0.707107,0.965926/
DATA PI6/0.52359877/
C POINTS FOR 6 POINT GAUSS QUADRATURE ON (-1,1) WITH WEIGHT FUNCTION
1/SQRT(1-U**2). WEIGHTS ARE PI/6
INDEX=NDEX-1
DO 2 I=1,300
AT(I,1,1)=0.0
2 AW(I,1,1)=0.0
4 CS=ST-SF
BT=BF+DS*CT54M
DO 12 L=1,2
5 DO 12 I=1,6
GO TO (6,7),L
6 SG=SF*V(I)
XM=1.0+SG*CT32P
B =1.0+SG*CT32M
RS=SF
GO TO 8
7 SG=ST-DS*V(I)**2
RS=2.0*DS*V(I)
SG1=SG-SF

```

F0RC

```

XM=BMF+SG1*CT54P
B = BF+SG1*CT54M
8 RS = RS*PI6*H(I)*SQRT(ST**2-SG**2)
D0 9 K=1,10
AS(K)=RS
9 RS=SG*RS
D0 12 J=1,6
Z=XM+B*U(J)
Z IS X
D0 12 L1=1,N
FAC=FXI (U(J),L1)
G0 T0 (23,24),L
23 D0 11 L2=1,6
D0 10 K=1,10
10 AW(L1,L2,K)=AW(L1,L2,K)+FAC*AS(K)
11 FAC=Z*FAC
G0 T0 12
24 D0 21 L2=1,6
D0 20 K=1,10
20 AT(L1,L2,K)=AT(L1,L2,K)+FAC*AS(K)
21 FAC=Z*FAC
12 CONTINUE
AREA=SF*(1.0+BF)+DS*(BF+BT)
D0 30 N1=1,NM0D
U0 30 N2=1,NM0D
FRC(1)=0.0
FRC(2)=0.0
I2 = 1
G0=1.0
D0 17 J=1,6
G=GO
G0=GO*BO
JL=7-J
D0 17 I = 1,JL
L3 = 1
F1=ZXY(I2,N2)*G
F2=ZZZ(I2,N2)*G

```

C

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00005430  
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00005450  
00005460  
00005470  
00005480  
00005490  
00005500  
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00005630  
00005640  
00005650  
00005660  
00005670  
00005680  
00005690  
00005700  
00005710  
00005720  
00005730  
00005740  
00005750  
00005760  
00005770  
00005780

F0RC

```
G=G*BO
IF (F1.EQ.0.0 .AND. F2.EQ.0.0) GO TO 17
Z=1.0
DO 19 K=1,M
  L2=K+J+IDEX
DO 18 L1=1,N
  F=F1*AW(L1,I,L2)+F2*AT(L1,I,L2)
  F=F*Z
DO 16 M1=1,2
  16 FRC(M1)=FRC(M1)+F*AP(M1,L3,N1)
  18 L3=L3+1
  19 Z=Z/ST
  17 I2 = 1 + I2
  F1=FRC(1)/AREA
  F2=FRC(2)/AREA
  F3=SQRT(F1**2+F2**2)
  F4=DOTS
  IF (F3.NE.0.0) F4=ATAN(F2,F1)*RAD
  30 WRITE (6,47) N1,N2,F1,F2,F3,F4
  47 FORMAT(1H02I6,1P3E19.5,0P1F16.4)
  RETURN
END
```

00005790  
00005800  
00005810  
00005820  
00005830  
00005840  
00005850  
00005860  
00005870  
00005880  
00005890  
00005900  
00005910  
00005920  
00005930  
00005940  
00005950  
00005960  
00005970  
00005980  
00005990  
00006000

# LSSURP

```

SUBROUTINE LSSURP(X,Y,Z,N,ND,NG,A,IND)
DIMENSION X(1),Y(1),Z(1),A(21),C(21,21),B(21)
DIMENSION G(21),F(21)
DIMENSION JLIM(6)

```

```

C      A POLYNOMIAL WITH THE COEFFICIENTS (A) IS FITTED BY LEAST SQUARES
C      TO THE VALUES (Z) AT THE POINTS (X,Y)
C      DEGREE ND IS GIVEN, MUST BE AT MOST 5
C      NUMBER OF POINTS N IS UNRESTRICTED
C      IND=0 FOR SUCCESSFUL FIT

```

```

IND=0
ILIM=ND+1
IF(ILIM.GT.6) ILIM=6
IJLIM=ILIM+1

```

```

DO 2 I=1,ILIM
3 JLIM(I)=IJLIM-I
3 NT=0

```

```

DO 4 I=1,ILIM
4 NT=NT+JLIM(I)

```

```

DO 5 I=1,NT

```

```

B(I)=0.0

```

```

DO 5 J=1,NT

```

```

5 C(J,I)=0.0

```

```

DO 8 K=1,N

```

```

YP=1.0

```

```

L=1

```

```

DO 7 I=1,ILIM

```

```

XYP=YP

```

```

JL=JLIM(I)

```

```

DO 6 J=1,JL

```

```

G(L)=XYP

```

```

XYP=X(K)*XYP

```

```

6 L=L+1

```

```

7 YP=Y(K)*YP

```

```

DO 8 I=1,NT

```

```

B(I)=B(I)+G(I)*Z(K)

```

```

DO 8 J=1,NT

```

```

8 C(J,I)=C(J,I)+G(J)*G(I)

```

```

00006020
00006030
00006040
00006050
00006060
00006070
00006080
00006090
00006100
00006110
00006120
00006130
00006140
00006150
00006160
00006170
00006180
00006190
00006200
00006210
00006220
00006230
00006240
00006250
00006260
00006270
00006280
00006290
00006300
00006310
00006320
00006330
00006340
00006350
00006360
00006370
00006380

```

LSSURP

```

00006390
00006400
00006410
00006420
00006430
00006440
00006450
00006460
00006470
00006480
00006490
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00006520
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00006610
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00006640
00006650
00006660
00006670
00006680
00006690
00006700
00006710
00006720
00006730
00006740
00006750

      DO 10 I=1,NT
      F(I)=0.0
      DO 9 J=1,NT
      9 F(I)=AMAX1(F(I),ABS(C(J,I)))
      DO 10 J=1,NT
      10 C(J,I)=C(J,I)/F(I)
      K=MSIMER(21,NT,1,C,B)
      IF (K.EQ.1) GO TO 15
      11 DO 12 I=1,ILIM
      IP=ILIM+1-I
      IF (JLIM(IP)+IP.EQ.IJLIM) GO TO 13
      12 CONTINUE
      IJLIM=IJLIM-1
      IF (IJLIM.GT.1) GO TO 11
      IND =1
      RETURN
      13 JLIM(IP)=JLIM(IP)-1
      IF (JLIM(IP).EQ.0) ILIM=IP-1
      GO TO 3
      15 NG=NT
      K=1
      L=1
      DO 18 I=1,ILIM
      JL=JLIM(I)
      JM=JL+1
      JN=7-I
      DO '6 J=1,JL
      A(L)=B(K)/F(K)
      K=K+1
      16 L=L+1
      IF (JN.LE.JL) GO TO 18
      DO 17 J=JM,JN
      A(L)=0.0
      17 L=L+1
      18 CONTINUE
      IF (L.GT.21) RETURN
      DO 19 K=L,21

```

00006760  
00006770  
00006780

19 A(K)=0.0  
RETURN  
END

ZDZ

```

SUBROUTINE ZDZ(X,Y,N,A,Z,DZ)
DIMENSION C(7),X(300),Y(300),Z(300),DZ(300)
DIMENSION A(21)
DATA C/0.0,1.0,2.0,3.0,4.0,5.0,6.0/
FINDS VALUES OF Z,DZ/DX AT THE COLLOCATION POINTS
DO 50 IN = 1,N
  Z(IN) = 0.0
  DZ(IN) = 0.0
  YP = 1.0
  NG = 1
  DO 20 I=1,6
    JL=7-I
    XYP = YP
    DO 10 J = 1,JL
      Z(IN) = Z(IN) + A(NG)*XYP
      XYP = X(IN)*XYP
      IF(J.EQ.1) GO TO 5
      DZ(IN) = DZ(IN) + C(J)*A(NG)*XYP
      XYQ = X(IN)*XYQ
      GO TO 10
    XYQ = YP
    NG = NG + 1
  20 YP = Y(IN)*YP
  50 CONTINUE
  WRITE (6,101)
  101 FORMAT(1H0,17X,1HX,20X,1HY,20X,1HZ,20X,2HDZ)
  DO 100 I = 1,N
    100 WRITE (6,105) X(I),Y(I),Z(I),DZ(I)
  105 FORMAT(1H ,10X,1PE15.4,5X,1PE15.4,6X,1PE15.4,7X,1PE15.4)
  RETURN
END

```

00006800  
00006810  
00006820  
00006830  
00006835  
00006840  
00006850  
00006860  
00006870  
00006880  
00006890  
00006900  
00006910  
00006920  
00006930  
00006940  
00006950  
00006960  
00006970  
00006980  
00006990  
00007000  
00007010  
00007020  
00007030  
00007040  
00007050  
00007060  
00007070  
00007080  
00007090

CNSIS

BLOCK DATA  
COMMON /CONST/ PI,PIOVR2,RAD,ZI(10)  
USEFUL CONSTANTS  
DATA PI/3.14159265/,PIOVR2/1.57079633/,RAD/57.2957795/  
DATA ZI/1.,2.,3.,4.,5.,6.,7.,8.,9.,10./  
END

00007110  
00007120  
00007125  
00007130  
00007140  
00007150

C

```

C      FUNCTION FXI(U1,K)
      EVALUATES THE KTH CHORDWISE PRESSURE MODE AT U1
      U=U1
      IF (1-K) 2,4,8
      2  FXI=1.0-U*U
      IF (K-5) 3,5,8
      3  IF (K-3) 8,5,7
      4  FXI=1.0-U
      GO TO 8
      5  FXI=-2.0*U*FXI
      IF (3-K) 6,8,8
      6  FXI=FXI*(4.0*U*U-2.0)
      GO TO 8
      7  FXI=FXI*(3.0-4.0*FXI)
      8  RETURN
      END
00007170
00007175
00007180
00007190
00007200
00007210
00007220
00007230
00007240
00007250
00007260
00007270
00007280
00007290
00007300
00007310

```



K=MSIMER(N,L,LB,A,B)

SIMR

|      |          |
|------|----------|
| STD  | A12+2    |
| STD  | A20      |
| STD  | A22      |
| STD  | A25+1    |
| STD  | A25+2    |
| STD  | A33      |
| STD  | A36+1    |
| STD  | A36+2    |
| SXD  | A52,1    |
| TXI  | *+1,1,1  |
| SCD  | A32,1    |
| TXI  | *+1,1,-2 |
| SCD  | A22+1,1  |
| CLA* | 5,4      |
| PAX  | 0,7      |
| SXA  | 1A6-1,7  |
| SXA  | A14,7    |
| SXA  | 1A21-1,7 |
| SXA  | A26,7    |
| SXA  | A37,7    |
| CLA* | 4,4      |
| PAX  | 0,1      |
| TXL  | E1,1,0   |
| TXH  | E1,1,**  |
| SXA  | A2,1     |
| SXA  | A5-1,1   |
| SXD  | A9,1     |
| SXD  | A12,1    |
| TXI  | *+1,1,-1 |
| SXD  | A7,1     |
| SXD  | A36,1    |
| SXD  | A18,1    |
| SXD  | A38-1,1  |
| SXD  | A21-1,1  |
| SXD  | A23,1    |
| SXD  | A25,1    |
| SXD  | A31,1    |

A52

|          |
|----------|
| 00050380 |
| 00050390 |
| 00050400 |
| 00050410 |
| 00050420 |
| 00050430 |
| 00050440 |
| 00050450 |
| 00050460 |
| 00050470 |
| 00050480 |
| 00050490 |
| 00050500 |
| 00050510 |
| 00050520 |
| 00050530 |
| 00050540 |
| 00050550 |
| 00050560 |
| 00050570 |
| 00050580 |
| 00050590 |
| 00050600 |
| 00050610 |
| 00050620 |
| 00050630 |
| 00050640 |
| 00050650 |
| 00050660 |
| 00050670 |
| 00050680 |
| 00050690 |
| 00050700 |
| 00050710 |
| 00050720 |
| 00050730 |
| 00050740 |

K=MSIMER(N,L,LB,A,B)

|      |     |             |  |  |  |
|------|-----|-------------|--|--|--|
| SIMR |     |             |  |  |  |
|      | SCD | A51,1       |  |  |  |
|      | SCD | A51+1,1     |  |  |  |
|      | TXI | *+1,1,1     |  |  |  |
|      | CLA | 6,4         |  |  |  |
|      | PAC | 0,3         |  |  |  |
|      | CLA | 7,4         |  |  |  |
|      | PAC | 0,5         |  |  |  |
| A51  | TXI | *+1,3, -L+1 |  |  |  |
| *    | TXI | *+1,5, -L+1 |  |  |  |
| *    |     |             |  |  |  |
| A2   |     |             |  |  |  |
|      | AXT | L,2         |  |  |  |
|      | PXA | 0,3         |  |  |  |
|      | PAX | 0,6         |  |  |  |
|      | PAX | 0,7         |  |  |  |
|      | PXA | 0,0         |  |  |  |
| A3   | LDQ | A,6         |  |  |  |
|      | LRS | 0           |  |  |  |
|      | TLQ | *+2         |  |  |  |
|      | XCA |             |  |  |  |
|      | TNX | A4,2,1      |  |  |  |
|      | TXI | A3,6, -N    |  |  |  |
| A4   | TZE | E1          |  |  |  |
|      | STG | T           |  |  |  |
|      | CLA | =1.0        |  |  |  |
|      | FDP | T           |  |  |  |
|      | STQ | T           |  |  |  |
| A5   | AXT | L,2         |  |  |  |
|      | FMP | A,7         |  |  |  |
|      | STG | A,7         |  |  |  |
|      | LDQ | T           |  |  |  |
|      | TNX | A6,2,1      |  |  |  |
|      | TXI | A5,7, -N    |  |  |  |
| A6   | PXA | 0,5         |  |  |  |
|      | PAX | 0,6         |  |  |  |
|      | AXT | LB,4        |  |  |  |

NORMALIZATION OF ROWS

00050750  
00050760  
00050770  
00050780  
00050790  
00050800  
00050810  
00050820  
00050830  
00050840  
00050850  
00050860  
00050870  
00050880  
00050890  
00050900  
00050910  
00050920  
00050930  
00050940  
00050950  
00050960  
00050970  
00050980  
00050990  
00051000  
00051010  
00051020  
00051030  
00051040  
00051050  
00051060  
00051070  
00051080  
00051090  
00051100  
00051110

K=MSIMER(N,L,LB,A,B)

SIMR

1A6

FMP B,6  
STG B,6  
LDQ T  
TNX 2A6,4,1  
TXI 1A6,6,-N  
TNX A7,1,1  
TXI \*\*1,3,1  
TXI A2,5,1  
TXH B7,2,L -1

A7

\*  
\*  
\*

SEARCH FOR MAXIMUM PIVOT IN COLUMN

PXA 0,2  
PAX 0,1  
PXA 0,3  
PAX 0,6  
PXA 0,0  
LDQ A,6  
LRS 0  
TLQ \*\*3

A8

XCA A10,1  
SXA \*\*1,1,1  
TXI \*\*2,1,L  
TXH A8,6,-1  
TXI TOL  
LDQ \*\*2  
TLQ E3  
TRA \*\*1  
AXT \*\*1,2  
SXD A17,1,\*\*

A9

A10

\*\*\*

ROW INTERCHANGE

PXA 0,3  
PAX 0,6  
PAX 0,7

00051120  
00051130  
00051140  
00051150  
00051160  
00051170  
00051180  
00051190  
00051200  
00051210  
00051220  
00051230  
00051240  
00051250  
00051260  
00051270  
00051280  
00051290  
00051300  
00051310  
00051320  
00051330  
00051340  
00051350  
00051360  
00051370  
00051380  
00051390  
00051400  
00051410  
00051420  
00051430  
00051440  
00051450  
00051460  
00051470  
00051480

K=MSIMER(N,L,LB,A,B)

SIMR

|     |     |            |          |
|-----|-----|------------|----------|
|     | SCD | *+1,1      | 00051490 |
|     | TXI | *+1,7,**   | 00051500 |
|     | PXA | 0,2        | 00051510 |
|     | PAX | 0,4        | 00051520 |
| A11 | CLA | A,6        | 00051530 |
|     | LDQ | A,7        | 00051540 |
|     | STQ | A,7        | 00051550 |
|     | STQ | A,6        | 00051560 |
|     | TXI | *+1,4,1    | 00051570 |
| A12 | TXH | A13,4,L    | 00051580 |
|     | TXI | *+1,6, -N  | 00051590 |
|     | TXI | A11,7, -N  | 00051600 |
| A13 | PXA | 0,5        | 00051610 |
|     | PAX | 0,6        | 00051620 |
|     | PAX | 0,7        | 00051630 |
| A14 | AXT | LB,4       | 00051640 |
|     | SCD | *+1,1      | 00051650 |
| A15 | TXI | *+1,6,**   | 00051660 |
|     | CLA | B,7        | 00051670 |
|     | LDQ | B,6        | 00051680 |
|     | STQ | B,6        | 00051690 |
|     | STQ | B,7        | 00051700 |
|     | TNX | A17,4,1    | 00051710 |
| A16 | TXI | *+1,6, -N  | 00051720 |
|     | TXI | A15,7, -N  | 00051730 |
| *   |     |            | 00051740 |
| *   |     |            | 00051750 |
| *   |     |            | 00051760 |
| A17 | CLA | =1.0       | 00051770 |
|     | FDP | A,3        | 00051780 |
|     | STQ | AM         | 00051790 |
| A18 | TXH | A21,2,L -1 | 00051800 |
|     | PXA | 0,2        | 00051810 |
|     | PAX | 0,4        | 00051820 |
|     | PXA | 0,3        | 00051830 |
|     | PAX | 0,6        | 00051840 |
| A20 | TXI | *+1,6, -N  | 00051850 |

DIVISION OF ROW BY PIVOT

K=MSIMER(N,L,LB,A,B)

SIMR

|      |     |             |          |
|------|-----|-------------|----------|
|      | LDQ | A,6         | 00051860 |
|      | FMP | AM          | 00051870 |
|      | STG | A,6         | 00051880 |
|      | TXI | *+1,4,1     | 00051890 |
|      | TXL | A20,4,L -1  | 00051900 |
| A21  | PXA | 0,5         | 00051910 |
|      | PAX | 0,6         | 00051920 |
|      | AXT | LB,4        | 00051930 |
| 1A21 | LDQ | B,6         | 00051940 |
|      | FMP | AM          | 00051950 |
|      | STG | B,6         | 00051960 |
|      | TNX | *+2,4,1     | 00051970 |
| 2A21 | TXI | 1A21,6, -N  | 00051980 |
| *    |     |             | 00051990 |
| *    |     |             | 00052000 |
| *    |     |             | 00052010 |
|      |     |             | 00052020 |
|      |     |             | 00052030 |
|      |     |             | 00052040 |
|      |     |             | 00052050 |
|      |     |             | 00052060 |
|      |     |             | 00052070 |
|      |     |             | 00052080 |
|      |     |             | 00052090 |
|      |     |             | 00052100 |
|      |     |             | 00052110 |
|      |     |             | 00052120 |
| A22  | TXI | *+1,7, -N   | 00052130 |
|      | TXI | *+1,3, -N+1 | 00052140 |
|      | SXA | A28,7       | 00052150 |
| A23  | TXH | A26,2,L -1  | 00052160 |
|      | SXA | A27,3       | 00052170 |
| A24  | LDQ | A,6         | 00052180 |
|      | FMP | A,7         | 00052190 |
|      | CHS |             | 00052200 |
|      | FAD | A,3         | 00052210 |
|      | STG | A,3         | 00052220 |

ROW REDUCTION

K=MSIMER(N,L,LB,A,B)

SIMR

|      |     |             |          |
|------|-----|-------------|----------|
| A25  | TXI | **1,4,1     | 00052230 |
|      | TXH | A27,4,L -1  | 00052240 |
|      | TXI | **1,3, -N   | 00052250 |
|      | TXI | A24,7, -N   | 00052260 |
| A27  | AXT | **3         | 00052270 |
| A26  | AXT | LB,4        | 00052280 |
|      | AXT | **7         | 00052290 |
|      | TXI | **1,7,1     | 00052300 |
|      | SXA | **2,7       | 00052310 |
| 1A26 | LDQ | A,6         | 00052320 |
|      | FMP | B,5         | 00052330 |
|      | CHS |             | 00052340 |
|      | FAD | B,7         | 00052350 |
|      | STG | B,7         | 00052360 |
|      | TNX | 3A26,4,1    | 00052370 |
| 2A26 | TXI | **1,5, -N   | 00052380 |
|      | TXI | 1A26,7, -N  | 00052390 |
| 3A26 | AXT | **5         | 00052400 |
|      | TNX | A29,1,1     | 00052410 |
| A28  | AXT | **7         | 00052420 |
|      | PXA | 0,2         | 00052430 |
|      | PAX | 0,4         | 00052440 |
|      | TXI | **1,3,1     | 00052450 |
|      | TXI | A23,6,1     | 00052460 |
| A29  | AXT | **3         | 00052470 |
| A31  | TXH | A43,2,L -1  | 00052480 |
|      | PXA | 0,2         | 00052490 |
|      | PAX | 0,1         | 00052500 |
|      | PAX | 0,4         | 00052510 |
|      | PXA | 0,3         | 00052520 |
|      | PAX | 0,6         | 00052530 |
|      | PAX | 0,7         | 00052540 |
| A32  | TXI | **1,3, -N-1 | 00052550 |
|      | TXI | **1,6,-1    | 00052560 |
| A33  | TXI | **1,7, -N   | 00052570 |
|      | SXA | A37+1,5     | 00052580 |
|      | SXA | A41,5       | 00052590 |

K=MSIMER(N,L,LB,A,B)

SIMR

|      |     |            |          |
|------|-----|------------|----------|
| A34  | SXA | A40,3      | 00052600 |
| A35  | SXA | A39,7      | 00052610 |
|      | SXA | A38,3      | 00052620 |
|      | LDQ | A,6        | 00052630 |
|      | FMP | A,7        | 00052640 |
|      | CH  |            | 00052650 |
|      | FAL | A,3        | 00052660 |
|      | STG | A,3        | 00052670 |
|      | TXI | *+1,4,1    | 00052680 |
| A36  | TXH | A37,4,L -1 | 00052690 |
|      | TXI | *+1,3, -N  | 00052700 |
|      | TXI | A35,7, -N  | 00052710 |
| A37  | AXT | LB,4       | 00052720 |
|      | AXT | **7        | 00052730 |
|      | TXI | *+1,7,-1   | 00052740 |
|      | SXA | *-2,7      | 00052750 |
| 1A37 | LDQ | A,6        | 00052760 |
|      | FMP | B,5        | 00052770 |
|      | CHS |            | 00052780 |
|      | FAD | B,7        | 00052790 |
|      | SYG | B,7        | 00052800 |
|      | INX | A41,4,1    | 00052810 |
| 2A37 | TXI | *+1,5, -N  | 00052820 |
|      | TXI | 1A37,7, -N | 00052830 |
| A41  | AYT | **5        | 00052840 |
|      | TXI | *+1,1,1    | 00052850 |
|      | TXH | A40,1,L -1 | 00052860 |
| A38  | AXT | **3        | 00052870 |
| A39  | AXT | **7        | 00052880 |
|      | PXA | 0,2        | 00052890 |
|      | PAX | 0,4        | 00052900 |
|      | TXI | *+1,3,-1   | 00052910 |
|      | TXI | A34,6,-1   | 00052920 |
| A40  | AXT | **3        | 00052930 |
|      | TXI | *+1,5,-1   | 00052940 |
|      | TXI | A7,2,1     | 00052950 |
| A45  | CLA | =1         | 00052960 |

K=MSIMER(N,L,LB,A,B)

SIMR

00052970  
00052980  
00052990  
00053000  
00053010  
00053020  
00053030  
00053040  
00053050  
00053060  
00053070  
00053080  
00053090  
00053100  
00053110  
00053120  
00053130  
00053140  
00053150  
00053160  
00053170  
00053180  
00053190  
00053200  
00053210  
00053220  
00053230

TRA MSIMER+1

BRANCH FOR LAST ROW

CLA A,3  
SSP  
LDQ TOL  
TLQ AL7

ERROR BRANCHES

CLA =2  
TRA MSIMER+1  
CLA =3  
TRA MSIMER+1

STORAGE

GCT 151400000000  
EQU E.1  
EQU E.2  
EQU 0  
EQU 0  
EQU 0  
EQU 0  
EQU 0  
EQU 0  
END

\*  
\*  
\*  
B7  
  
\*  
\*  
\*  
E3  
E1  
\*  
\*  
\*  
TOL  
AM  
T  
A  
B  
L  
N  
LB



K=MSIMEC(N,L,LB,A,B)

SIMC

STD 2A37+1  
 STD A12+2  
 STD A20  
 STD A22  
 STD A25+1  
 STD A25+2  
 STD A33  
 STD A36+1  
 STD A36+2  
 STD A52+1  
 TXI \*+1+1,2  
 SCD A32+1  
 TXI \*+1+1,-4  
 SCD A22+1,1  
 CLA\* 5,4  
 PAX 0,7  
 SXA 1A6-1,7  
 SXA A14,7  
 SXA 1A21-1,7  
 SXA A26,7  
 SXA A37,7  
 CLA\* 4,4  
 ALS 1  
 PAX 0,1  
 TXL E1,1,1  
 TXH E1,1,\*  
 SXA A2,1  
 SXA A5-1,1  
 SXD A9,1  
 SXD A12,1  
 TXI \*+1,1,-2  
 SXD A7,1  
 SXD A36,1  
 SXD A18,1  
 SXD A38-1,1  
 SXD A21-1,1  
 SXD A23,1

A52

00053620  
 00053630  
 00053640  
 00053650  
 00053660  
 00053670  
 00053680  
 00053690  
 00053700  
 00053710  
 00053720  
 00053730  
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 00053780  
 00053790  
 00053800  
 00053810  
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 00053890  
 00053900  
 00053910  
 00053920  
 00053930  
 00053940  
 00053950  
 00053960  
 00053970  
 00053980

K=MSIMEC(N,L,LB,A,B)

SIMC

00053990  
00054000  
00054010  
00054020  
00054030  
00054040  
00054050  
00054060  
00054070  
00054080  
00054090  
00054100  
00054110  
00054120  
00054130  
00054140  
00054150  
00054160  
00054170  
00054180  
00054190  
00054200  
00054210  
00054220  
00054230  
00054240  
00054250  
00054260  
00054270  
00054280  
00054290  
00054300  
00054310  
00054320  
00054330  
00054340  
00054350

A25,1  
A31,1  
A51,1  
A51+1,1  
\*+1,1,2  
6,4  
0,3  
7,4  
0,5  
\*+1,3,2L -2  
\*+1,5,2L -2  
NORMALIZATION OF ROWS  
AXT 2L,2  
PXA 0,3  
PAX 0,6  
PAX 0,7  
PXA 0,0  
LDQ A,6  
LRS 0  
TLQ \*+2  
XCA A+1,6  
LDQ 0  
LRS \*+2  
XCA A4,2,2  
TNX A3,6,2N  
TXI E1  
TZE T  
STG =1.0  
CLA T  
FDP T  
STQ 2L,2  
AXT A,7  
FMP A,7  
STG A,7

A51

\*

\*

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A2

A3

A4

A5

K=MSIMEC(N,L,LB,A,B)

SIMC

|     |     |            |
|-----|-----|------------|
|     | LDQ | T          |
|     | FMP | A+1,7      |
|     | STQ | A+1,7      |
|     | LDQ | T          |
|     | TNX | A6,2,2     |
|     | TXI | A5,7,2N    |
| A6  | PXA | O,5        |
|     | PAX | O,6        |
|     | AXT | LB,4       |
| 1A6 | FMP | B,6        |
|     | STQ | B,6        |
|     | LDQ | T          |
|     | FMP | B+1,6      |
|     | STQ | B+1,6      |
|     | LDQ | T          |
|     | TNX | 2A6,4,1    |
| 2A6 | TXI | 1A6,6,2N   |
|     | TNX | A7,1,2     |
|     | TXI | **1,3,2    |
|     | TXI | A2,5,2     |
| A7  | TXH | B7,2,2L -2 |
| *   |     |            |
| *   |     |            |
| *   |     |            |

SEARCH FOR MAXIMUM PIVOT IN COLUMN

|  |     |       |
|--|-----|-------|
|  | PXA | O,2   |
|  | PAX | O,1   |
|  | PXA | O,3   |
|  | PAX | O,6   |
|  | PXA | O,0   |
|  | LDQ | A,6   |
|  | LRS | O     |
|  | TLQ | **+3  |
|  | XCA |       |
|  | SXA | A10,1 |
|  | LDQ | A+1,6 |
|  | LRS | O     |
|  | TLQ | **+3  |

A8

00054360  
00054370  
00054380  
00054390  
00054400  
00054410  
00054420  
00054430  
00054440  
00054450  
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00054630  
00054640  
00054650  
00054660  
00054670  
00054680  
00054690  
00054700  
00054710  
00054720

K=MSIMEC(N,L,LB,A,B)

SIMC

|     |     |          |          |
|-----|-----|----------|----------|
| A9  | XCA | A10,1    | 00054730 |
|     | SXA | **1,1,2  | 00054740 |
|     | TXI | **2,1,2L | 00054750 |
|     | TXH | A8,6,-2  | 00054760 |
|     | TXI | T0L      | 00054770 |
|     | LDQ | **2      | 00054780 |
|     | TLQ | E3       | 00054790 |
|     | TRA | **1      | 00054800 |
| A10 | AXT | **1,2    | 00054810 |
|     | SXD | A17,1,** | 00054820 |
|     | TNX |          | 00054830 |
| *   |     |          | 00054840 |
| *   |     |          | 00054850 |
| *   |     |          | 00054860 |
|     | PXA | 0,3      | 00054870 |
|     | PAX | 0,6      | 00054880 |
|     | PAX | 0,7      | 00054890 |
|     | SCD | **1,1    | 00054900 |
|     | TXI | **1,7,** | 00054910 |
|     | PXA | 0,2      | 00054920 |
| A11 | PAX | 0,4      | 00054930 |
|     | CLA | A,6      | 00054940 |
|     | LDQ | A,7      | 00054950 |
|     | STQ | A,7      | 00054960 |
|     | STQ | A,6      | 00054970 |
|     | CLA | A+1,6    | 00054980 |
|     | LDQ | A+1,7    | 00054990 |
|     | STQ | A+1,7    | 00055000 |
|     | STQ | A+1,6    | 00055010 |
| A12 | TXI | **1,4,2  | 00055020 |
|     | TXH | A13,4,2L | 00055030 |
|     | TXI | **1,6,2N | 00055040 |
|     | TXI | A11,7,2N | 00055050 |
| A13 | PXA | 0,5      | 00055060 |
|     | PAX | 0,6      | 00055070 |
|     | PAX | 0,7      | 00055080 |
| A14 | AXT | LB,4     | 00055090 |

ROW INTERCHANGE

K=MSIMEC(N,L,LB,A,B)

| SIMC |     |  |             |  |          |
|------|-----|--|-------------|--|----------|
|      | SCD |  | ++1,1       |  | 00055100 |
|      | TXI |  | ++1,6,**    |  | 00055110 |
| A15  | CLA |  | B,7         |  | 00055120 |
|      | LDQ |  | B,6         |  | 00055130 |
|      | STG |  | B,6         |  | 00055140 |
|      | STQ |  | B,7         |  | 00055150 |
|      | CLA |  | B+1,7       |  | 00055160 |
|      | LDQ |  | B+1,6       |  | 00055170 |
|      | STG |  | B+1,6       |  | 00055180 |
|      | STQ |  | B+1,7       |  | 00055190 |
| A16  | TNX |  | A17,4,1     |  | 00055200 |
|      | TXI |  | ++1,6,2N    |  | 00055210 |
|      | TXI |  | A15,7,2N    |  | 00055220 |
| *    |     |  |             |  | 00055230 |
| *    |     |  |             |  | 00055240 |
| *    |     |  |             |  | 00055250 |
| A17  | NZT |  | A,3         |  | 00055260 |
|      | TRA |  | B1          |  | 00055270 |
|      | LDQ |  | A,3         |  | 00055280 |
|      | FMP |  | A,3         |  | 00055290 |
|      | STG |  | T           |  | 00055300 |
|      | LDQ |  | A+1,3       |  | 00055310 |
|      | FMP |  | A+1,3       |  | 00055320 |
|      | FAD |  | T           |  | 00055330 |
|      | STG |  | T           |  | 00055340 |
|      | CLA |  | A,3         |  | 00055350 |
|      | FDP |  | T           |  | 00055360 |
|      | STQ |  | AM          |  | 00055370 |
|      | CLA |  | A+1,3       |  | 00055380 |
|      | FDP |  | T           |  | 00055390 |
|      | STQ |  | AN          |  | 00055400 |
| A18  | TXH |  | A21,2,2L -2 |  | 00055410 |
|      | PXA |  | O,2         |  | 00055420 |
|      | PAX |  | O,4         |  | 00055430 |
|      | PXA |  | O,3         |  | 00055440 |
|      | PAX |  | O,6         |  | 00055450 |
| A20  | TXI |  | ++1,6,2N    |  | 00055460 |

DIVISION OF ROW BY PIVOT

K=MSIMEC(N,L,LB,A,B)

SIMC

|     |             |          |
|-----|-------------|----------|
| LDQ | A,6         | 00055470 |
| FMP | AN          | 00055480 |
| STG | T           | 00055490 |
| LDQ | A+1,6       | 00055500 |
| FMP | AM          | 00055510 |
| FSB | T           | 00055520 |
| LDQ | A+1,6       | 00055530 |
| STG | A+1,6       | 00055540 |
| FMP | AN          | 00055550 |
| STG | T           | 00055560 |
| LDQ | A,6         | 00055570 |
| FMP | AM          | 00055580 |
| FAD | T           | 00055590 |
| STG | A,6         | 00055600 |
| TXI | *+1,4,2     | 00055610 |
| TXL | A20,4,2L -2 | 00055620 |
| PXA | 0,5         | 00055630 |
| PAX | 0,6         | 00055640 |
| AXT | LB,4        | 00055650 |
| LDQ | B,6         | 00055660 |
| FMP | AN          | 00055670 |
| STG | T           | 00055680 |
| LDQ | B+1,6       | 00055690 |
| FMP | AM          | 00055700 |
| FSB | T           | 00055710 |
| LDQ | B+1,6       | 00055720 |
| STG | B+1,6       | 00055730 |
| FMP | AN          | 00055740 |
| STG | T           | 00055750 |
| LDQ | B,6         | 00055760 |
| FMP | AM          | 00055770 |
| FAD | T           | 00055780 |
| STG | B,6         | 00055790 |
| TXN | *+2,4,1     | 00055800 |
| TXI | 1A21,6,2N   | 00055810 |
|     |             | 00055820 |
|     |             | 00055830 |

ROW REDUCTION

2A21

\* \*

K=MSIMEC(N,L,LB,A,B)

SIMC

|     |     |             |          |
|-----|-----|-------------|----------|
|     | PXA | 0,2         | 00055840 |
|     | PAX | 0,1         | 00055850 |
|     | PAX | 0,4         | 00055860 |
|     | TNX | A31,1,2     | 00055870 |
|     | PXA | 0,3         | 00055880 |
|     | PAX | 0,6         | 00055890 |
|     | PAX | 0,7         | 00055900 |
|     | STA | A29         | 00055910 |
|     | SXA | A26+1,5     | 00055920 |
|     | SXA | 3A26,5      | 00055930 |
|     | TXI | *+1,6,2     | 00055940 |
| A22 | TXI | *+1,7,2N    | 00055950 |
|     | TXI | *+1,3,2N -2 | 00055960 |
|     | SXA | A28,7       | 00055970 |
| A23 | TXH | A26,2,2L -2 | 00055980 |
|     | SXA | A27,3       | 00055990 |
| A24 | LDQ | A,6         | 00056000 |
|     | FMP | A,7         | 00056010 |
|     | STG | T           | 00056020 |
|     | LDQ | A+1,6       | 00056030 |
|     | FMP | A+1,7       | 00056040 |
|     | FSB | T           | 00056050 |
|     | FAD | A,3         | 00056060 |
|     | STG | A,3         | 00056070 |
|     | LDQ | A,6         | 00056080 |
|     | FMP | A+1,7       | 00056090 |
|     | STG | T           | 00056100 |
|     | LDQ | A+1,6       | 00056110 |
|     | FMP | A,7         | 00056120 |
|     | FAD | T           | 00056130 |
|     | CHS |             | 00056140 |
|     | FAD | A+1,3       | 00056150 |
|     | STG | A+1,3       | 00056160 |
|     | TXI | *+1,4,2     | 00056170 |
| A25 | TXH | A27,4,2L -2 | 00056180 |
|     | TXI | *+1,3,2N    | 00056190 |
|     |     |             | 00056200 |

K=MSIMEC(N,L,LB,A,B)

SIMC

|      |     |             |
|------|-----|-------------|
| A27  | TXI | A24,7,2N    |
| A26  | AXT | **3         |
|      | AXT | LB,4        |
|      | AXT | **7         |
|      | TXI | **1,7,2     |
|      | SXA | *-2,7       |
| 1A26 | LDQ | A,6         |
|      | FMP | B,5         |
|      | STG | T           |
|      | LDQ | A+1,6       |
|      | FMP | B+1,5       |
|      | FSB | T           |
|      | FAD | B,7         |
|      | STG | B,7         |
|      | LDQ | A,6         |
|      | FMP | B+1,5       |
|      | STG | T           |
|      | LDQ | A+1,6       |
|      | FMP | B,5         |
|      | FAD | T           |
|      | CHS |             |
|      | FAJ |             |
|      | STG | B+1,7       |
|      | FNX | B+1,7       |
| 2A26 | TXI | 3A26,4,1    |
|      | TXI | **1,5,2N    |
| 3A26 | AXT | 1A26,7,2N   |
|      | TNX | **5         |
| A28  | AXT | A29,1,2     |
|      | PXA | **7         |
|      | PAX | O,2         |
|      | TXI | O,4         |
|      | TXI | **1,3,2     |
|      | AXT | A23,6,2     |
| A29  | TXH | **3         |
| A31  | PXA | A43,2,2L -2 |
|      | PAX | O,2         |
|      |     | O,1         |

00056210  
00056220  
00056230  
00056240  
00056250  
00056260  
00056270  
00056280  
00056290  
00056300  
00056310  
00056320  
00056330  
00056340  
00056350  
00056360  
00056370  
00056380  
00056390  
00056400  
00056410  
00056420  
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00056480  
00056490  
00056500  
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00056520  
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00056540  
00056550  
00056560  
00056570

K=MSIMEC(N,L,LB,A,B)

SIMC

|     |     |             |
|-----|-----|-------------|
|     | PAX | 0,4         |
|     | PXA | 0,3         |
|     | PAX | 0,6         |
|     | PAX | 0,7         |
| A32 | TXI | *+1,3,2N +2 |
|     | TXI | *+1,6,-2    |
| A33 | TXI | *+1,7,2N    |
|     | SXA | A37+1,5     |
|     | SXA | A41,5       |
|     | SXA | A40,3       |
|     | SXA | A39,7       |
| A34 | SXA | A38,3       |
| A35 | LDQ | A,6         |
|     | FMP | A,7         |
|     | STG | T           |
|     | LDQ | A+1,6       |
|     | FMP | A+1,7       |
|     | FSB | T           |
|     | FAD | A,3         |
|     | STG | A,3         |
|     | LDQ | A,6         |
|     | FMP | A+1,7       |
|     | STG | T           |
|     | LDQ | A+1,6       |
|     | FMP | A,7         |
|     | FAD | T           |
|     | CHS | A+1,3       |
|     | FAD | A+1,3       |
|     | STG | *+1,4,2     |
| A36 | TXI | A37,4,2L -2 |
|     | TXH | *+1,3,2N    |
|     | TXI | A35,7,2N    |
|     | TXI | LB,4        |
| A37 | AXT | *+7         |
|     | AXT | *+1,7,-2    |
|     | TXI | *-2,7       |
|     | SXA |             |

00056580  
00056590  
00056600  
00056610  
00056620  
00056630  
00056640  
00056650  
00056660  
00056670  
00056680  
00056690  
00056700  
00056710  
00056720  
00056730  
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00056780  
00056790  
00056800  
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00056860  
00056870  
00056880  
00056890  
00056900  
00056910  
00056920  
00056930  
00056940

K=MSIMEC(N,L,LB,A,B)

SIMC

|      |     |             |          |
|------|-----|-------------|----------|
| 1A37 | LDQ | A,6         | 00056950 |
|      | FMP | B,5         | 00056960 |
|      | STG | T           | 00056970 |
|      | LDQ | A+1,6       | 00056980 |
|      | FMP | B+1,5       | 00056990 |
|      | FSB | T           | 00057000 |
|      | FAD | B,7         | 00057010 |
|      | STG | B,7         | 00057020 |
|      | LDQ | A,6         | 00057030 |
|      | FMP | B+1,5       | 00057040 |
|      | STG | T           | 00057050 |
|      | LDQ | A+1,6       | 00057060 |
|      | FMP | B,5         | 00057070 |
|      | FAD | T           | 00057080 |
|      | CHS |             | 00057090 |
|      | FAD | B+1,7       | 00057100 |
|      | STG | B+1,7       | 00057110 |
|      | TNX | A41,4,1     | 00057120 |
| 2A37 | TXI | **+1,5,2N   | 00057130 |
|      | TXI | 1A37,7,2N   | 00057140 |
| A41  | AXT | **+5        | 00057150 |
|      | TXI | **+1,1,2    | 00057160 |
|      | TXH | A40,1,2L -2 | 00057170 |
| A38  | AXT | **+3        | 00057180 |
| A39  | AXT | **+7        | 00057190 |
|      | PXA | 0,2         | 00057200 |
|      | PAX | 0,4         | 00057210 |
|      | TXI | **+1,3,-2   | 00057220 |
|      | TXI | A34,6,-2    | 00057230 |
| A40  | AXT | **+3        | 00057240 |
|      | TXI | **+1,5,-2   | 00057250 |
|      | TXI | A7,2,2      | 00057260 |
| A43  | CLA | =1          | 00057270 |
|      | TRA | MSIMEC+1    | 00057280 |
| B1   | STZ | AM          | 00057290 |
|      | CLA | =1.0        | 00057300 |
|      | FDP | A+1,3       | 00057310 |

K=MSIMEC(N,L,LB,A,B)

SIMC

STQ AN  
TRA A18

BRANCH FOR LAST ROW

CLA A,3  
SSP  
LOQ TOL  
TLQ A17+2  
CLA A+1,3  
SSP  
TLQ A17

ERROR BRANCHES

CLA =2  
TRA MSIMEC+1  
CLA =3  
TRA MSIMEC+1

STORAGE

GCT 151400000000

EQU 0  
EQU 0  
EQU 0  
EQU 0  
EQU 0  
END

00057320  
00057330  
00057340  
00057350  
00057360  
00057370  
00057380  
00057390  
00057400  
00057410  
00057420  
00057430  
00057440  
00057450  
00057460  
00057470  
00057480  
00057490  
00057500  
00057510  
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00057570  
00057580  
00057590  
00057600  
00057610  
00057620  
00057630



# N.A.A. SUBROUTINE LIBRARY

| BESEL | 700    |    |          |
|-------|--------|----|----------|
| TSX   | B,4    |    | BESE2690 |
| XCA   |        |    | BESE2700 |
| FMP   | FRE+18 |    | BESE2710 |
| FAD   | FRE+16 |    | BESE2720 |
| STG   | FRE+16 |    | BESE2730 |
| CAS   | FRE+17 |    | BESE2740 |
| TRA   | K2     |    | BESE2750 |
| TRA   | K3     |    | BESE2760 |
| STG   | FRE+17 | K2 | BESE2770 |
| CLA   | FRE+2  |    | BESE2780 |
| FAD   | FL01   |    | BESE2790 |
| STG   | FRE+2  |    | BESE2800 |
| TRA   | K1     |    | BESE2810 |
| STG   | FRE+18 | K3 | BESE2820 |
| CLM   |        |    | BESE2830 |
| SLW   | FRE+2  |    | BESE2840 |
| SLW   | FRE+16 |    | BESE2850 |
| CLA   | FL01   |    | BESE2860 |
| STG   | FRE+15 |    | BESE2870 |
| TSX   | C,4    | K4 | BESE2880 |
| XCA   |        |    | BESE2890 |
| FMP   | FRE+15 |    | BESE2900 |
| FAD   | FRE+16 |    | BESE2910 |
| STG   | FRE+16 |    | BESE2920 |
| CLS   | FRE+15 |    | BESE2930 |
| STG   | FRE+15 |    | BESE2940 |
| CLA   | FRE+2  |    | BESE2950 |
| FAD   | FL01   |    | BESE2960 |
| STG   | FRE+2  |    | BESE2970 |
| FSB   | FRE+1  |    | BESE2980 |
| THI   | K4     |    | BESE2990 |
| CLA   | FRE+14 | K6 | BESE3000 |
| LBT   |        |    | BESE3010 |
| TRA   | K7     |    | BESE3020 |
| CLA   | FL01   |    | BESE3030 |
| TRA   | K8     |    | BESE3040 |
| CLS   | FL01   | K7 | BESE3050 |

# N.A.A. SUBROUTINE LIBRARY

BESEL 700

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K8 XCA      FRE+18
FMP         FRE+16
FAD         FL02
FDP         FRE+21
STQ         FRE+21
CLA         ESC
TRA         ASYMPOTOTIC EXPANSION OF KNX
          FRE
          FL088
          KAI
          CLS
          FRE
          FRE+24
          EXP(FRE+24)
          FRE+21
          CALCULATE SQUARE RT OF PI/2X
          FRE
          FRE
          FRE+18
          PI
          FRE+18
          FRE+19
          SQRT(FRE+19)
          XCA
          FKP
          STQ
          CLA
          STQ
          STQ
          LDQ
          FMP
          STQ
          STQ
          LDQ
          FMP
          XCA
          FMP
          FL04

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BESE3060  
BESE3070  
BESE3080  
BESE3090  
BESE3100  
BESE3110  
BESE3120  
BESE3130  
BESE3140  
BESE3150  
BESE3160  
BESE3170  
BESE3180  
BESE3190  
BESE3200  
BESE3210  
BESE3220  
BESE3230  
BESE3240  
BESE3250  
BESE3260  
BESE3270  
BESE3280  
BESE3290  
BESE3300  
BESE3310  
BESE3320  
BESE3330  
BESE3340  
BESE3350  
BESE3360  
BESE3370  
BESE3380  
BESE3390  
BESE3400  
BESE3410  
BESE3420

# N.A.A. SUBROUTINE LIBRARY

|       |      |     |                 |          |
|-------|------|-----|-----------------|----------|
| BESEL | 700  | STG | FRE+7           | BESE3430 |
|       |      | FBS | FLG1            | BESE3440 |
|       |      | FDP | FRE+11          | BESE3450 |
|       |      | STQ | FRE+8           | BESE3460 |
|       |      | CLA | FRE+8           | BESE3470 |
|       | KA4  | FAD | FRE+16          | BESE3480 |
|       |      | STG | FRE+16          | BESE3490 |
|       |      | CAS | FRE+17          | BESE3500 |
|       |      | TRA | KA5             | BESE3510 |
|       |      | TRA | KA6             | BESE3520 |
|       | KA5  | STG | FRE+17          | BESE3530 |
|       |      | CLA | FRE+3           | BESE3540 |
|       |      | FAD | FLG1            | BESE3550 |
|       |      | STG | FRE+3           | BESE3560 |
|       |      | TSX | D,4             | BESE3570 |
|       |      | TRA | KA4             | BESE3580 |
|       | KA6  | FAD | FLG1            | BESE3590 |
|       |      | XCA |                 | BESE3600 |
|       |      | FMP | FRE+21          | BESE3610 |
|       |      | STG | FRE+19          | BESE3620 |
|       |      | CLS | FRE             | BESE3630 |
|       |      | FAD | FLG88           | BESE3640 |
|       |      | TMI | KA7             | BESE3650 |
|       |      | CLA | FRE+19          | BESE3660 |
|       |      | TRA | ESC             | BESE3670 |
|       | KA7  | LDQ | FRE+19          | BESE3680 |
|       | KA8  | FMP | EM88            | BESE3690 |
|       |      | TRA | ESC             | BESE3700 |
|       | *    |     | FACTGRIAL       | BESE3710 |
|       | *    |     | M IN ACC        | BESE3720 |
|       | *    |     | LEAVE M/ IN ACC | BESE3730 |
|       | FACT | TNZ | FACT+3          | BESE3740 |
|       |      | CLA | FLG1            | BESE3750 |
|       |      | TRA | 1,4             | BESE3760 |
|       |      | STG | FRE+4           | BESE3770 |
|       |      | STG | FRE+5           | BESE3780 |
|       |      | CLA | FRE+4           | BESE3790 |

# N.A.A. SUBROUTINE LIBRARY

|       |     |      |                        |          |
|-------|-----|------|------------------------|----------|
| BESEL | 700 | FSB  | FL01                   | BESE3800 |
|       |     | TZE  | FACT+12                | BESE3810 |
|       |     | STG  | FRE+4                  | BESE3820 |
|       |     | LDQ  | FRE+5                  | BESE3830 |
|       |     | FMP  | FRE+4                  | BESE3840 |
|       |     | TRA  | FACT+4                 | BESE3850 |
|       |     | CLA  | FRE+5                  | BESE3860 |
|       |     | TRA  | 1,4                    | BESE3870 |
|       |     |      | SUBR.                  | BESE3880 |
|       |     |      | X/2,N+2S/S/N+S/        | BESE3890 |
|       |     |      | X/2,N AND S IN STORAGE | BESE3900 |
|       |     |      | RESULT LEFT IN ACC     | BESE3910 |
|       |     |      | FRE+10,4               | BESE3920 |
|       |     | SXD  | FRE+1                  | BESE3930 |
|       |     | CLA  | FRE+2                  | BESE3940 |
|       |     | FAD  | FRE+7                  | BESE3950 |
|       |     | STG  | FRE+2                  | BESE3960 |
|       |     | FAD  | FRE+8                  | BESE3970 |
|       |     | STG  | FRE+2                  | BESE3980 |
|       |     | CLA  | FACT,4                 | BESE3990 |
|       |     | TSX  | FRE+9                  | BESE4000 |
|       |     | STG  | FRE+7                  | BESE4010 |
|       |     | CLA  | FRE+7                  | BESE4020 |
|       |     | TSX  | FACT,4                 | BESE4030 |
|       |     | XCA  |                        | BESE4040 |
|       |     | FMP  | FRE+9                  | BESE4050 |
|       |     | STG  | FRE+9                  | BESE4060 |
|       |     | CLA  | FRE+8                  | BESE4070 |
|       |     | TNZ  | A1                     | BESE4080 |
|       |     | CLA  | FL01                   | BESE4090 |
|       |     | TRA  | A2                     | BESE4100 |
|       |     | LDQ  | FRE+6                  | BESE4110 |
|       |     | FMP  | FRE+8                  | BESE4120 |
|       |     | STG  | FRE+24                 | BESE4130 |
|       |     | CALL | EXP(FRE+24)            | BESE4140 |
|       |     | FDP  | FRE+9                  | BESE4150 |
|       |     | STQ  | FRE+9                  | BESE4160 |
|       |     | CLA  | FRE+9                  |          |

\* \* \* \* A  
 A1  
 A2

# N.A.A. SUBROUTINE LIBRARY

| BESEL | 700                                       |  |          |
|-------|---|--|----------|
| LXD   | FRE+10,4                                  |  | BESE4170 |
| TRA   | 1,4                                       |  | BESE4180 |
|       | PARENTHEICAL SUBR                         |  | BESE4190 |
|       | X/2,N,AND S                               |  | BESE4200 |
|       | LEAVE RESULT IN ACC.                      |  | BESE4210 |
|       | FRE+10,4                                  |  | BESE4220 |
|       | FRE+2                                     |  | BESE4230 |
|       | B1,4                                      |  | BESE4240 |
|       |   |  | BESE4250 |
|       | FRE+8                                     |  | BESE4260 |
|       | FRE+1                                     |  | BESE4270 |
|       | FRE+2                                     |  | BESE4280 |
|       | B1,4                                      |  | BESE4290 |
|       |   |  | BESE4300 |
|       | FRE+8                                     |  | BESE4310 |
|       | 2GAM                                      |  | BESE4320 |
|       | FRE+8                                     |  | BESE4330 |
|       | FRE+6                                     |  | BESE4340 |
|       | FLQ2                                      |  | BESE4350 |
|       | FRE+8                                     |  | BESE4360 |
|       | FRE+10,4                                  |  | BESE4370 |
|       | 1,4                                       |  | BESE4380 |
|       | SUMMATION OF 1/Y FROM R EQUALS 1 TO LIMIT |  | BESE4390 |
|       | FRE+3                                     |  | BESE4400 |
|       | 1,4                                       |  | BESE4410 |
|       |   |  | BESE4420 |
|       | FRE+9                                     |  | BESE4430 |
|       | FLQ1                                      |  | BESE4440 |
|       | FRE+3                                     |  | BESE4450 |
|       | FRE+11                                    |  | BESE4460 |
|       | FRE+11                                    |  | BESE4470 |
|       | FRE+9                                     |  | BESE4480 |
|       | FRE+9                                     |  | BESE4490 |
|       | FRE+3                                     |  | BESE4500 |
|       | FLQ1                                      |  | BESE4510 |
|       | B1+15                                     |  | BESE4520 |
|       | FRE+3                                     |  | BESE4530 |

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| BESEL | 700   |  |          |
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| TRA   | B1+4  |  | BESE4540 |
| CLA   | FRE+9   |  | BESE4550 |
| TRA   | 1,4   |  | BESE4560 |
|       | SUBROUTINE FOR LAST TERM OF SERIES OF YN AND KN |  | BESE4570 |
|       | X/2,N AND S IN STORAGE                          |  | BESE4580 |
|       | LEAVE RESULT IN ACCUMULATOR                     |  | BESE4590 |
|       | FRE+1   |  | BESE4600 |
|       | FL01  |  | BESE4610 |
|       | C3  |  | BESE4620 |
|       | 1,4   |  | BESE4630 |
|       | FRE+10,4  |  | BESE4640 |
|       | FRE+2   |  | BESE4650 |
|       | FACT,4  |  | BESE4660 |
|       | FRE+7   |  | BESE4670 |
|       | FRE+1   |  | BESE4680 |
|       | FRE+2   |  | BESE4690 |
|       | FL01  |  | BESE4700 |
|       | FACT,4  |  | BESE4710 |
|       | FRE+7   |  | BESE4720 |
|       | FRE+7   |  | BESE4730 |
|       | FRE+2   |  | BESE4740 |
|       | FRE+2   |  | BESE4750 |
|       | FRE+1   |  | BESE4760 |
|       | C1  |  | BESE4770 |
|       | FL01  |  | BESE4780 |
|       | C2  |  | BESE4790 |
|       | FRE+8   |  | BESE4800 |
|       | FRE+6   |  | BESE4810 |
|       | FRE+8   |  | BESE4820 |
|       | FRE+24  |  | BESE4830 |
|       | EXP(FRE+24)                                     |  | BESE4840 |
|       |   |  | BESE4850 |
|       | FRE+7   |  | BESE4860 |
|       | FRE+10,4  |  | BESE4870 |
|       | 1,4   |  | BESE4880 |
|       | LEAVE RESULT IN ACC.                            |  | BESE4890 |
|       |   |  | BESE4900 |

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LDQ

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FDP

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CONSTANTS FOR BESSEL FUNCTIONS

\* KSW0

DEC

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|  | 2b. GROUP<br><div style="text-align: center;">N/A</div>  |                       |
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| 1. DESCRIPTIVE NOTES (Type of report and inclusive dates)<br>Final Report  |  |                       |
| 5. AUTHOR(S) (Last name, first name, initial)<br><br>Vivian, H. T.<br>Andrew, L. V.  |  |                       |
| 6. REPORT DATE<br>May 1965   | 7a. TOTAL NO. OF PAGES<br>110  | 7b. NO. OF REFS<br>13 |
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| d.   |  |                       |
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| 11. SUPPLEMENTARY NOTES  | 12. SPONSORING MILITARY ACTIVITY<br>Air Force Flight Dynamics Laboratory (FDDS)<br>Wright-Patterson AFB, Ohio 45433              |                       |
| 13. ABSTRACT<br><br>Equations for pressure distributions and generalized aerodynamic forces are derived for a thin nonplanar lifting surface in simple harmonic motion at subsonic speeds. A digital computer program, written in Fortran IV, is also presented. The computer program will generate up to a ten by ten matrix of generalized aerodynamic forces when given data for the geometry of a planar lifting surface with a folded planar tip, the flight Mach number, the reduced frequency of motion, and some control constants. Control surface deflections are not accounted for in this study. The kernel function method given by Watkins, Runyan, and Woolston (Reference 1), which relates the pressure distribution to the downwash on a planar lifting surface, has been extended and applied to a nonplanar lifting surface. Hsu's technique (Reference 4) of employing Gaussian quadrature formulas is used when integrating the product of the kernel function and the lift function over the planform area. Recommendations are made to extend the method to account for blunted leading edges and the accompanying airfoil thickness and to account for control surface deflections. |  |                       |